Hilbert Transform :

Fourier Transform → Evaluating the Frq content of a Signal.

→ Mathematical basis → Analyzing & designing "Frq Selective Filters."

↳ Seperation of signals on the basis of frq Content.

Another Method → Separating the signal based on phase selectivity ⇒ "HILBERT TRANSFORM."

FT → Frq domain analysis.
HT → Time domain analysis.

Definition :

① → If phase angle of all the components of a given signal are shifted by ±90° then the resulting function of Time is HILBERT TRANSFORM of the signal

H.T. of $x(t)$
$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau} \rightarrow ①$$

Inverse H.T.
$$\boxed{x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{t-\tau}\, d\tau} \rightarrow ②$$

From definition, $\hat{x}(t)$ → convolution of $x(t)$ with time fun $\left(\frac{1}{\pi t}\right)$

$$\therefore \hat{x}(t) = x(t) * \frac{1}{\pi t} \longrightarrow ③$$

W.K.T. Convolution in Time domain is Product in Frq domain

$\therefore FT\{x(t)\} = X(f)$

$FT\left\{\frac{1}{\pi t}\right\} = -j\,sgn(f)$

$$\boxed{sgn(f) = \begin{cases} 1 \ ; & f > 0 \\ 0 \ ; & f = 0 \\ -1 \ ; & f < 0 \end{cases}}$$

$\therefore$ F.T. of eqn ③ ⇒

$$\hat{X}(f) = X(f) \cdot [-j\,sgn(f)]$$

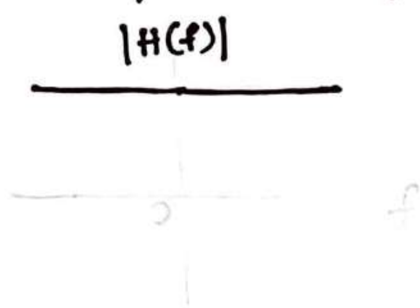
$$\therefore \hat{X}(f) = -j\,sgn(f) \cdot X(f) \longrightarrow ④$$

$$x(t) \rightarrow \boxed{F.T.} \xrightarrow{X(f)} \bigotimes_{\uparrow -j\,sgn(f)} \xrightarrow{\hat{X}(f)} \boxed{IFT} \rightarrow \hat{x}(t)$$

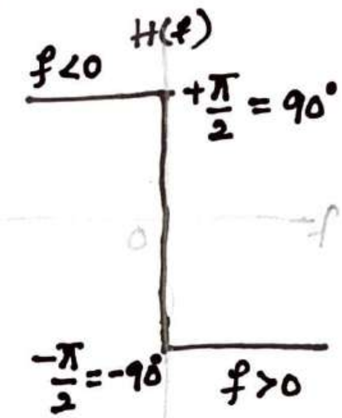
Two Port device

Two port device → produce phase shift

- -90° for all positive frq of i/p signal.
- $+90^\circ$ for all negative frq



(a) Amplitude



(b) Phase.

Applications:

→ Generation of single side Band (SSB) modulation.

→ Represent the bandpass signal Mathematically.

Hilbert Transform Pairs

Time function		Hilbert Transform
$m(t)\cos(2\pi f_c t)$	→	$m(t)\sin(2\pi f_c t)$
$m(t)\sin(2\pi f_c t)$	→	$-m(t)\cos(2\pi f_c t)$
$\cos(2\pi f_c t)$	→	$\sin(2\pi f_c t)$
$\sin(2\pi f_c t)$	→	$-\cos(2\pi f_c t)$
$\frac{\sin t}{t}$	→	$\frac{1-\cos t}{t}$
$\delta(t)$	→	$\frac{1}{\pi t}$
$\frac{1}{1+t^2}$	→	$\frac{1}{1+t^2}$
$\frac{1}{t}$	→	$-\pi\delta(t)$

Obtain the Hilbert Transform

ⓐ $x(t) = \cos(2\pi f_c t)$

$$\hat{x}(t) = \cos\left(2\pi f_c t - \frac{\pi}{2}\right)$$
$$= \cos[-(\pi/2 - 2\pi f_c t)]$$
$$= \cos[\pi/2 - 2\pi f_c t]$$

$$\boxed{\hat{x}(t) = \sin(2\pi f_c t)}$$

$\cos(-\theta) \downarrow \cos\theta$

$\cos(\pi/2 - \theta) \downarrow \sin\theta$

$\sin(-\theta) \rightarrow -\sin\theta$

$\sin(\pi/2 - \theta) \rightarrow \cos\theta$

ⓑ $x(t) = \cos 2\pi f t + \sin 2\pi f t$

$$\hat{x}(t) = \cos(2\pi f t - \pi/2) + \sin\left(2\pi f t - \frac{\pi}{2}\right)$$
$$= \cos[-(\pi/2 - 2\pi f t)] + \sin\left[-\left(\frac{\pi}{2} - 2\pi f t\right)\right]$$
$$= \cos(\pi/2 - 2\pi f t) - \sin(\pi/2 - 2\pi f t)$$

$$\boxed{\hat{x}(t) = \sin(2\pi f t) - \cos(2\pi f t)}$$

ⓒ $x(t) = e^{-j2\pi f t}$

$$= \cos(2\pi f t) - j\sin(2\pi f t)$$

$$\hat{x}(t) = \cos\left(2\pi f t - \frac{\pi}{2}\right) - j\sin\left(2\pi f t - \frac{\pi}{2}\right)$$
$$= \cos[-(\pi/2 - 2\pi f t)] - j\sin\left[-\left(\frac{\pi}{2} - 2\pi f t\right)\right]$$
$$= \cos(\pi/2 - 2\pi f t) + j\sin(\pi/2 - 2\pi f t)$$
$$= \sin(2\pi f t) + j\cos(2\pi f t)$$

$\sin\theta + j\cos\theta$

$j[\cos\theta - j\sin\theta]$

$j\cos\theta - j^2\sin\theta$

$j\cos\theta + \sin\theta$

$je^{-j\theta}$

$$\boxed{\hat{x}(t) = j e^{-j2\pi f t}}$$

Properties of Hilbert Transform:

$x(t) \longrightarrow$ assumed to be real valued signal.

Property 1: A signal $x(t)$ & it's H.T. $\hat{x}(t)$ have the same magnetude spectrum.

$$|X(f)| = |\hat{X}(f)|$$

W.K.T $\hat{X}(f) = -j\,\mathrm{sgn}(f)\,X(f)$

$$|\hat{X}(f)| = |-j\,\mathrm{sgn}(f)\,X(f)|$$

$$|\hat{X}(f)| = |X(f)| \qquad \because |-j\,\mathrm{sgn}(f)| = 1$$

Property 2: If $\hat{x}(t)$ is H.T. of $x(t)$ then H.T. of $\hat{x}(t)$ is $-x(t)$

$$x(t) \longrightarrow \boxed{\text{H.T}} \xrightarrow{\hat{x}(t)} \boxed{\text{H.T.}} \longrightarrow -x(t)$$

$$\text{H.T} \times \text{H.T} \Rightarrow -j\,\mathrm{sgn}(f) \times -j\,\mathrm{sgn}(f)$$

$$= j^2\,\mathrm{sgn}^2(f) \qquad \because \mathrm{sgn}^2(f) = 1$$

$= -1$ .... all the values of 'f' $\quad j^2 = -1$

H.T. $\hat{x}(t)$ is $-x(t)$

Property 3: A signal $x(t)$ & its H.T. $\hat{x}(t)$ are orthogonal over entire time interval $(-\infty, \infty)$

i.e. $\displaystyle\int_{-\infty}^{\infty} x(t)\cdot\hat{x}(t)\,dt = 0$

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\cdot\hat{X}(-f)\,df.$$

but $\hat{X}(f) = -j\,\mathrm{sgn}(f)\,X(f)$

then $\hat{X}(-f) = -j\,\mathrm{sgn}(-f)\,X(-f)$

$$= j\,\mathrm{sgn}(f)\cdot X(-f)$$

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\,j\,\mathrm{sgn}(f)\cdot X(-f)\,df$$

$$= \int_{-\infty}^{\infty} j\,\mathrm{sgn}(f)\,|X(f)|^2\,df.$$

$\mathrm{sgn}(f) \rightarrow$ odd fun $\qquad |X(f)|^2 \rightarrow$ even fun

Integration of an odd fun over the range $-\infty$ to ∞ will be $\Rightarrow$ "ZERO"

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0$$

Pre Envelops:

$$\boxed{x_+(t) = x(t) + j\hat{x}(t)} \longrightarrow (1)$$

$j \times -j = -j^2 = -(-1)$

$x_+(t) \rightarrow$ Pre envelope for +ve freq

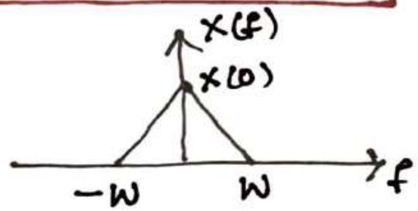
F.T. $\Rightarrow X_+(f) = X(f) + j[-j\,sgn(f)]X(f) \rightarrow (2)$

$= X(f) + sgn(f)X(f)$

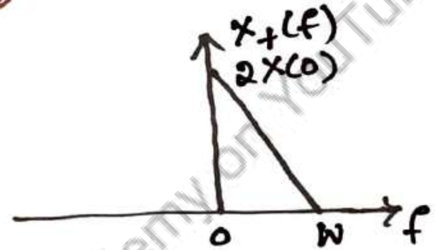
$X_+(f) = X(f)[1 + sgn(f)] \longrightarrow (3)$

$sgn(f) = \begin{cases} 1 ; f>0 \\ 0 ; f=0 \\ -1 ; f<0 \end{cases}$

$$\therefore \boxed{X_+(f) = \begin{cases} 2X(f) ; & f>0 \\ X(0) ; & f=0 \\ 0 ; & f<0 \end{cases}} \longrightarrow (4)$$



(a) Amplitude Spectrum of low pass signal



(b) Amplitude spectrum of Preenvelope for +ve freq's.

Pre envelope for -ve freq's

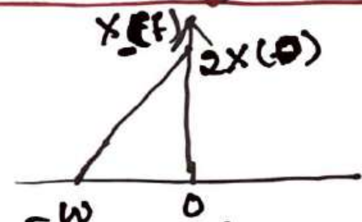
$$\boxed{x_-(t) = x(t) - j\hat{x}(t)} \longrightarrow (5)$$

F.T. $\Rightarrow X_-(f) = X(f) - j[-j\,sgn(f)]X(f)$

$= X(f) - sgn(f)X(f)$

$X_-(f) = X(f)[1 - sgn(f)] \rightarrow (6)$

$$\boxed{X_-(f) = \begin{cases} 0 ; & f>0 \\ X(0) ; & f=0 \\ 2X(f) ; & f<0 \end{cases}} \longrightarrow (7)$$



(c) Amplitude spectrum of Pre envelope for -ve freq's

Procedure to find Pre envelope:

1. determine the F.T. $X(f)$ of $x(t)$
2. Use $X_+(f) = \begin{cases} 2X(f) ; & f>0 \\ X(0) ; & f=0 \\ 0 ; & f<0 \end{cases}$ to find $X_+(f)$
3. Evaluate IFT of $X_+(f)$ to obtain $\boxed{x_+(t) = 2\int_0^{\infty} x(t)\, e^{j2\pi ft}\, dt}$

Complex Envelope of Band-Pass Signal:

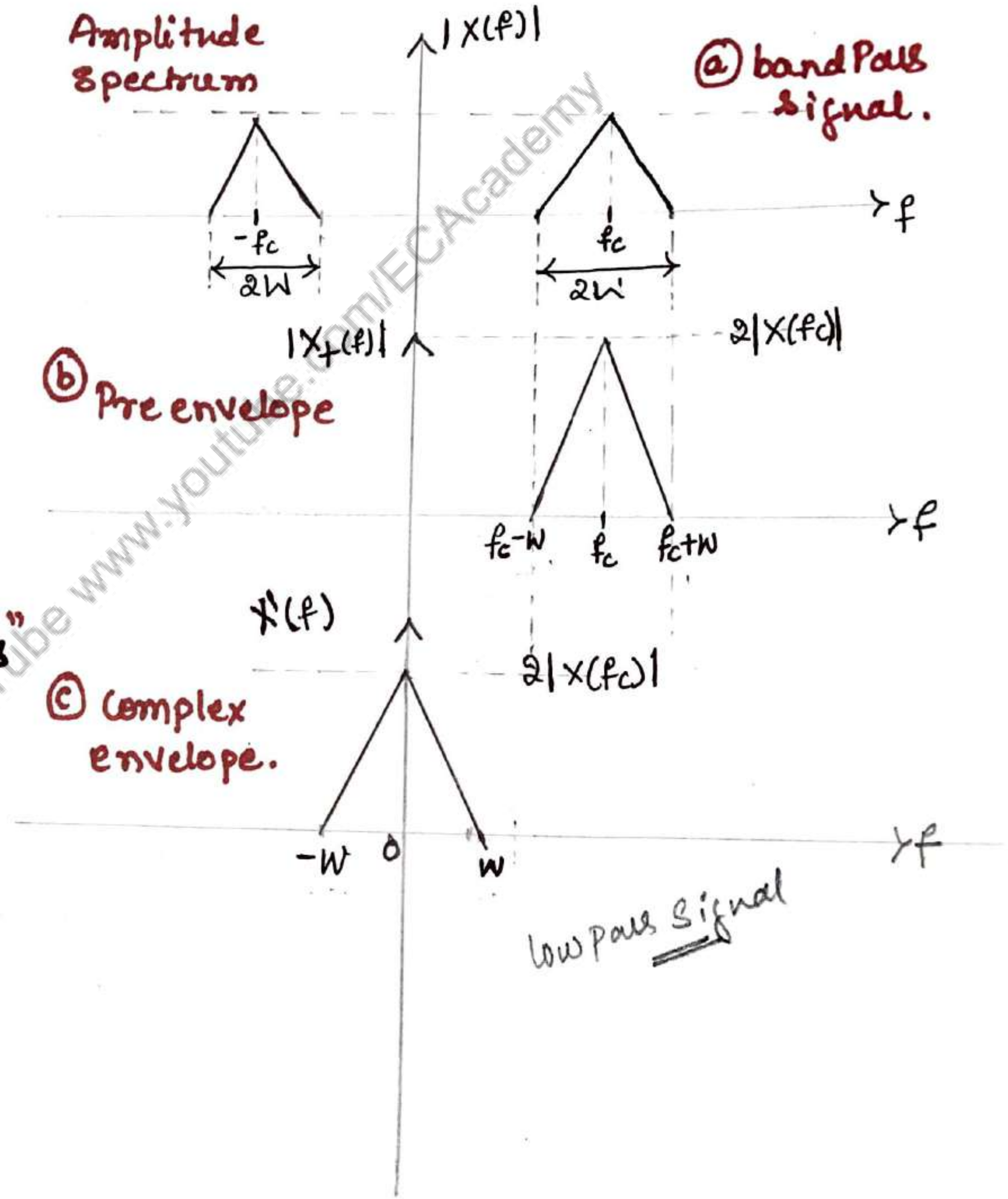
$x(t) \xrightarrow{F.T.} X(f) \rightarrow$ has a band of frq:ies of '2W' & Centered about frq: '$\pm f_c$' $\Rightarrow$ "Band Pass Signal"

$f_c \rightarrow$ Carrier frq:
↳ very high when compared to band width $2W \Rightarrow$ "Narrow band signals"

→ The pre envelope of such a narrow band signal $x(t)$ with its F.T. $X(f)$ centered about frq: $\pm f_c$ can be given as,

$$\boxed{x_+(t) = x'(t)\,e^{j2\pi ft}} \longrightarrow (1)$$

$x'(t) \longrightarrow$ complex envelope.



Canonical Representation of Bandpass Signals:

$x(t) \rightarrow$ real part of Pre envelope $x_+(t)$.

$$\therefore \quad x(t) = \mathrm{Re}\left[x'(t)\, e^{j2\pi f_c t}\right] \longrightarrow (1)$$

$x'(t) \rightarrow$ complex valued quantity.

$$\therefore \quad x'(t) = x_I(t) + j\,x_Q(t) \longrightarrow (2)$$

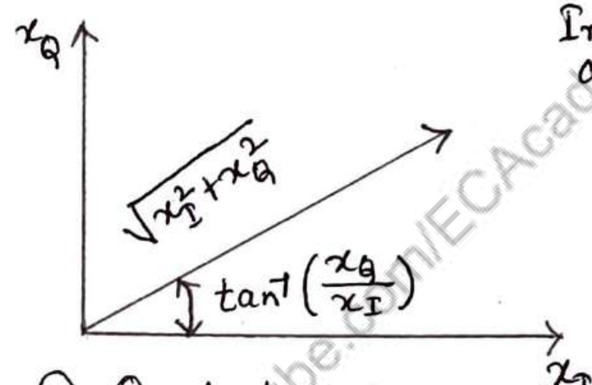
$x_I(t)$ & $x_Q(t) \rightarrow$ real valued lowpass fun.

using eqn (2), in canonical form

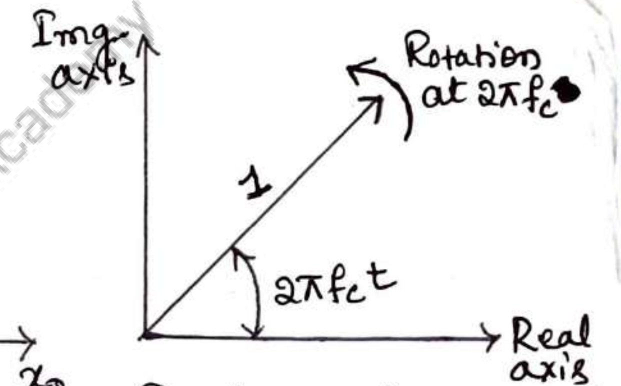
$$(1) \Rightarrow \boxed{x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t} \longrightarrow (3)$$

here $x_I(t) \rightarrow$ In phase componet

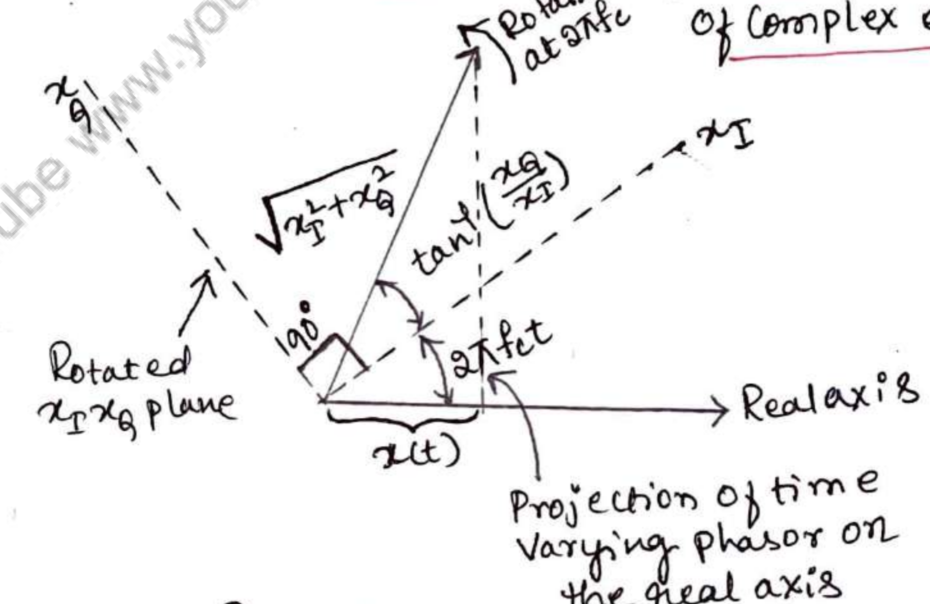
& $x_Q(t) \rightarrow$ Quadrature Component of the signal.



(a) Quadrature component



(b) Phasor representation of complex exponential



(c) Multiplication of complex envelope by complex exponential

$$x(t) = \mathrm{Re}\left[\left\{x_I(t) + j\,x_Q(t)\right\} e^{j2\pi f_c t}\right]$$

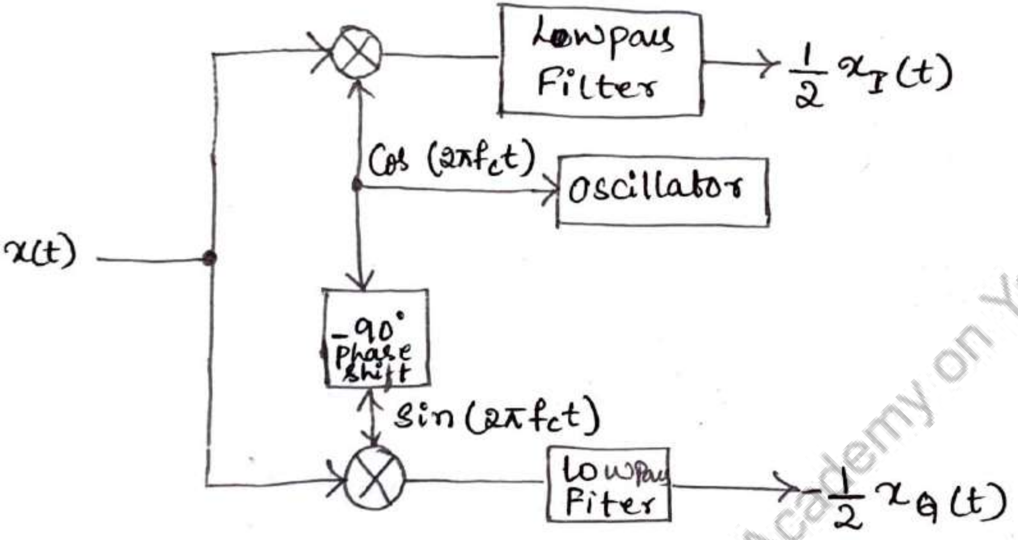
$$= \mathrm{Re}\left[\left\{x_I(t) + j\,x_Q(t)\right\} \cos(2\pi f_c t) + j\sin(2\pi f_c t)\right.$$

$$= \mathrm{Re}\left[\left\{ \underline{x_I(t)\cos(2\pi f_c t)} + j\,x_I(t)\sin(2\pi f_c t) + j\,x_Q(t)\cos(2\pi f_c t) - \underline{x_Q(t)\sin(2\pi f_c t)}\right]\right.$$

$$x(t) = x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)$$

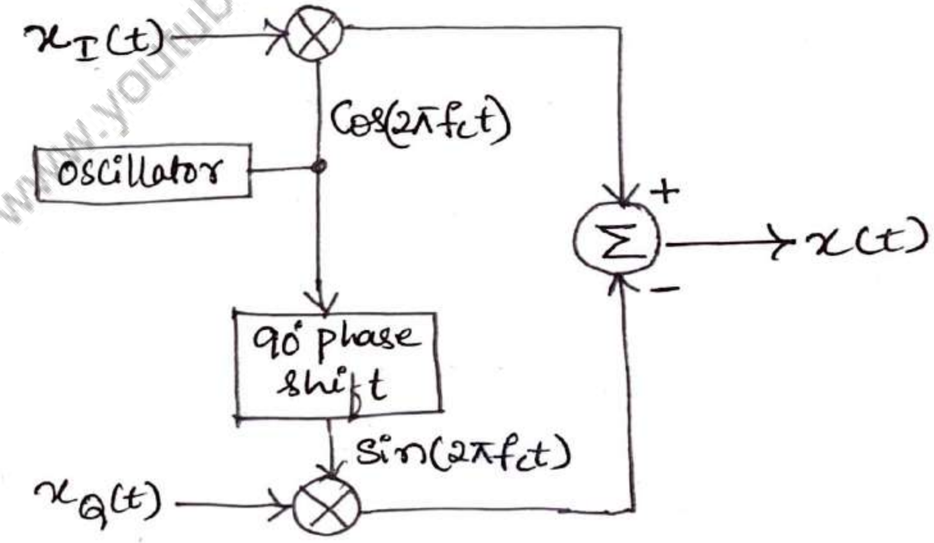
$x_I(t)$ & $x_Q(t)$ → lowpass signal with band limited to $-W \le f \le W$

→ These signals can be derived from band pass signal $x(t)$, as shown in fig (d).



(d) Derivation of $x_I(t)$
(Analizer)

→ Fig (e) shows the circuit to reconstruct $x(t)$ from $x_I(t)$ & $x_Q(t)$.



(e) Reconstruction of $x(t)$
(Synthesizer)

Polar representation of Bandpass signal.

In. Polar form, $\boxed{x'(t) = a(t)\, e^{j\phi(t)}}$

$x'(t) \rightarrow$ Complex envelope

$a(t)$ & $\phi(t) \rightarrow$ real valued lowpass fun^{s}

in polar form, the basedpass signal can be represented as,

$$\boxed{x(t) = a(t) \cos[2\pi f_c t + \phi(t)]}$$

$a(t) \rightarrow$ Natural Envelope (or) envelope of band pass signal $x(t)$

$\phi(t) \rightarrow$ the phase angle of the signal.

→ Three different envelopes to describe B.P.S. $x(t)$.

1. Pre-envelope:

Positive frq $\Rightarrow$ $\boxed{x_+(t) = x(t) + j\hat{x}(t)}$

F.T. using signum fun

$$x_+(t) = \begin{cases} 2X(f) \,;\, f>0 \\ X(0) \,;\, f=0 \\ 0 \,;\, f<0 \end{cases}$$

negative frq $\Rightarrow$ $\boxed{x_-(t) = x(t) - j\hat{x}(t)}$

F.T. $\Rightarrow$ $x_-(t) = \begin{cases} 0 \,;\, f>0 \\ X(0) \,;\, f=0 \\ 2X(f) \,;\, f<0 \end{cases}$

2. Complex envelope:

$\boxed{x'(t) = x_+(t)\, e^{-j2\pi f_c t}}$ $f_c \rightarrow$ Carrier frq

3. Natural Envelope:

$$a(t) = |x'(t)| = |x_+(t)|$$

relate $a(t)$ & $x_I(t)$, $x_Q(t)$

$$a(t) = \sqrt{x_I^2(t) + x_Q^2(t)}$$

$$\phi(t) = \tan^{-1}\left[\frac{x_Q(t)}{x_I(t)}\right]$$

and,

$$x_I(t) = a(t) \cos[\phi(t)]$$

$$x_Q(t) = a(t) \sin[\phi(t)]$$

Complex lowpass representation of Bandpass System:

signal $s(t)$ is applied to a linear time invariant s/m with impulse response $h(t)$ → bandpass impulse response

$h(t) \rightarrow h_I(t)$ & $h_Q(t)$

$$h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t) \longrightarrow ①$$

$h'(t) \rightarrow$ complex envelope
↳ lowpass fun

$$h'(t) = h_I(t) + j h_Q(t) \longrightarrow ②$$

$h(t)$ in terms of $h'(t)$ as,

$$h(t) = Re\left[h'(t)e^{j2\pi f_c t}\right] \longrightarrow ③$$

Let, $h'^*(t) \rightarrow$ complex conjugate of $h(t)$

$h(t)$ interms of $h'^*(t)$

$$h(t) = Re\left[h'^*(t)e^{-j2\pi f_c t}\right] \longrightarrow ④$$

Add eqn ③ & ④

$$2h(t) = h'(t)e^{j2\pi f_c t} + h'^*(t)e^{-j2\pi f_c t} \longrightarrow ⑤$$

Take F.T. of eqn ⑤

$$2H(f) = H'(f-f_c) + H'^*(-f-f_c) \longrightarrow ⑥$$

for real valued signal $H^*(f) = H(-f)$

$$⑥ \Rightarrow H'(f-f_c) = 2H(f)\ ;\ f>0 \longrightarrow ⑦$$

∴ the complex lowpass frequency response $H'(f)$ of linear time invarient s/m can be obtained by taking bandpass freq response $H(f)$ for positive frequencies by shifting it to origin & scaling the amplitude by 2

the complex lowpass freq response,

$$H'(f) = H'_I(f) + jH'_Q(f) \longrightarrow ⑧$$

here ; $H'_I(f) = \frac{1}{2}\left[H'(f) + H'^*(-f)\right]$

& $H'_Q(f) = \frac{1}{2j}\left[H'(f) - jH'^*(f)\right]$

∴ Complex impulse resonse $h(t)$ IFT of $H'(f)$

$$\Rightarrow \boxed{h'(t) = \int_{-\infty}^{\infty} H'(f)e^{j2\pi f}\,df}$$

Complex Representation of Bandpass Signals & Systems:

- modulated signal → $x(t)$ & impulse response → $h(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \rightarrow ①$$

Preenvelope as,

$$y(t) = \int_{-\infty}^{\infty} Re[h_+(\tau)] \cdot Re[x_+(t-\tau)]\, d\tau$$

$$= \frac{1}{2} Re\left[\int_{-\infty}^{\infty} h_+(\tau)\, x_+(t-\tau)\, d\tau\right]$$

$$= \frac{1}{2} Re\left[\int_{-\infty}^{\infty} h'(\tau)\, e^{j2\pi f_c \tau} \cdot x'(t-\tau) \cdot e^{j2\pi f_c (t-\tau)}\, d\tau\right]$$

$$= \frac{1}{2} Re\left[e^{j2\pi f_c t} \int_{-\infty}^{\infty} h'(\tau)\, x'(t-\tau)\, d\tau\right] \rightarrow ②$$

Let $y'(t) = \frac{1}{2}\int_{-\infty}^{\infty} h'(\tau)\, x'(t-\tau)\, d\tau \rightarrow ③$

$$\therefore y(t) = Re\left[y'(t)\, e^{j2\pi f_c t}\right] \rightarrow ④$$

Complex envelope of o/p signal $x'(t)$ in terms of phase & Quadrature components as,

$$y'(t) = y_I(t) + j\, y_Q(t)$$

$$\therefore 2y_I(t) = h_I(t) * x_I(t) - h_Q(t) * x_Q(t)$$

$$\& \quad 2y_Q(t) = h_Q(t) * x_I(t) + h_I(t) * x_Q(t)$$

Baseband shaping for Data Transmission:

→ Digital data can be transmitted directly without any modulation of carrier signal.
⇒ "Baseband signal Transmission".

→ Digital data ⇒ Various Electrical form.
1 → +ve amplitude 'A'
0 → -ve amplitude '-A'
Encoded waveform ⇒ '±A'.

→ These representations ⇒ Digital data formats, digital PAM signals or "Line code"

Line Codes:

→ Analog to digital signal ⇒ PCM, DM, DPCM etc... ⇒ "Line Codes"

→ Digital Symobls,

$$\boxed{x(t) = \sum a_n P(t - nD)} \longrightarrow (1)$$

here: a_n → Modulating Amplitude
n → n^{th} symbol in the message.
$P(t)$ → Carrier signal.
↳ It's pulses are modulated by 'a_n'

D → max duration allowed for carrier pulse.

→ Unmodulated $P(t)$ → rectangular pulse,

$$P(t) = 1 \,;\, t = 0 \qquad \longrightarrow (2)$$
$$\phantom{P(t) = {}} 0 \,;\, t = \pm D, \pm 2D$$

→ $x(t)$ → base band signal
→ recover $x(t)$ → Sample at fixed interval.
⇒ "DETECTION"

→ (1) ⇒ if $P(t) = 0$ then $x(t) = 0$ → no digital information present.

→ preferable to sample $x(t)$ when $P(t) = 0$
∴ $x(t)$ → sampled at $t = nD$; $n = 0, \pm 1, \pm 2 \ldots$

$P(t) = \text{rect}\left[\frac{t}{\tau}\right] \longrightarrow (3)$ Pulse to pulse interval ⇒ 'D'

∴ $\tau \le D$ The signalling rate $r = \frac{1}{D} \longrightarrow (4)$

if $D = \frac{1}{T_b}$ → duration of one bit. $\boxed{r = \frac{1}{T_b}} \longrightarrow (5)$

Unipolar RZ & NRZ:

RZ → Return to zero

NRZ → Not Return to zero

Polar → Polarity uni → one +ve

Waveform will have single polarity

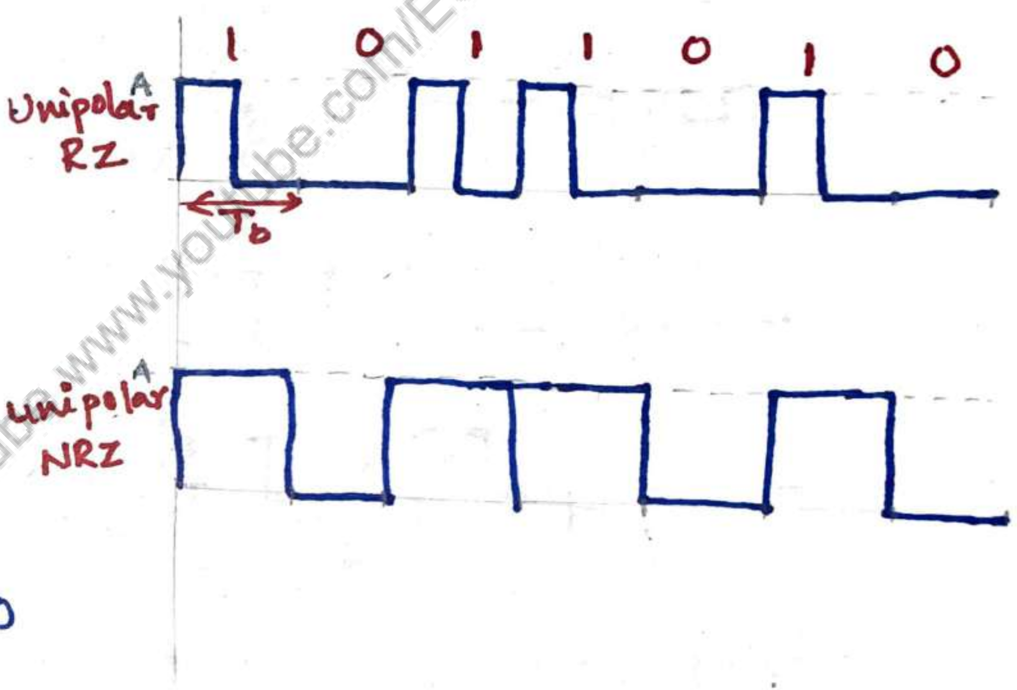
Unipolar RZ:

T_b → one bit duration.

Symbol 0 → 0 → complete T_b

Symbol 1 → A → for $T_b/2$ Period

& remaining $\frac{T_b}{2}$ period → 0



Unipolar NRZ:

Symbol 0 → 0 complete T_b

Symbol 1 → A complete T_b

Polar RZ | NRZ, Bipolar NRZ & spilt phase Manchester:

Polar RZ:

Symbol 0 → $-\frac{A}{2}$; $\frac{T_b}{2}$

0 ; $\frac{T_b}{2}$

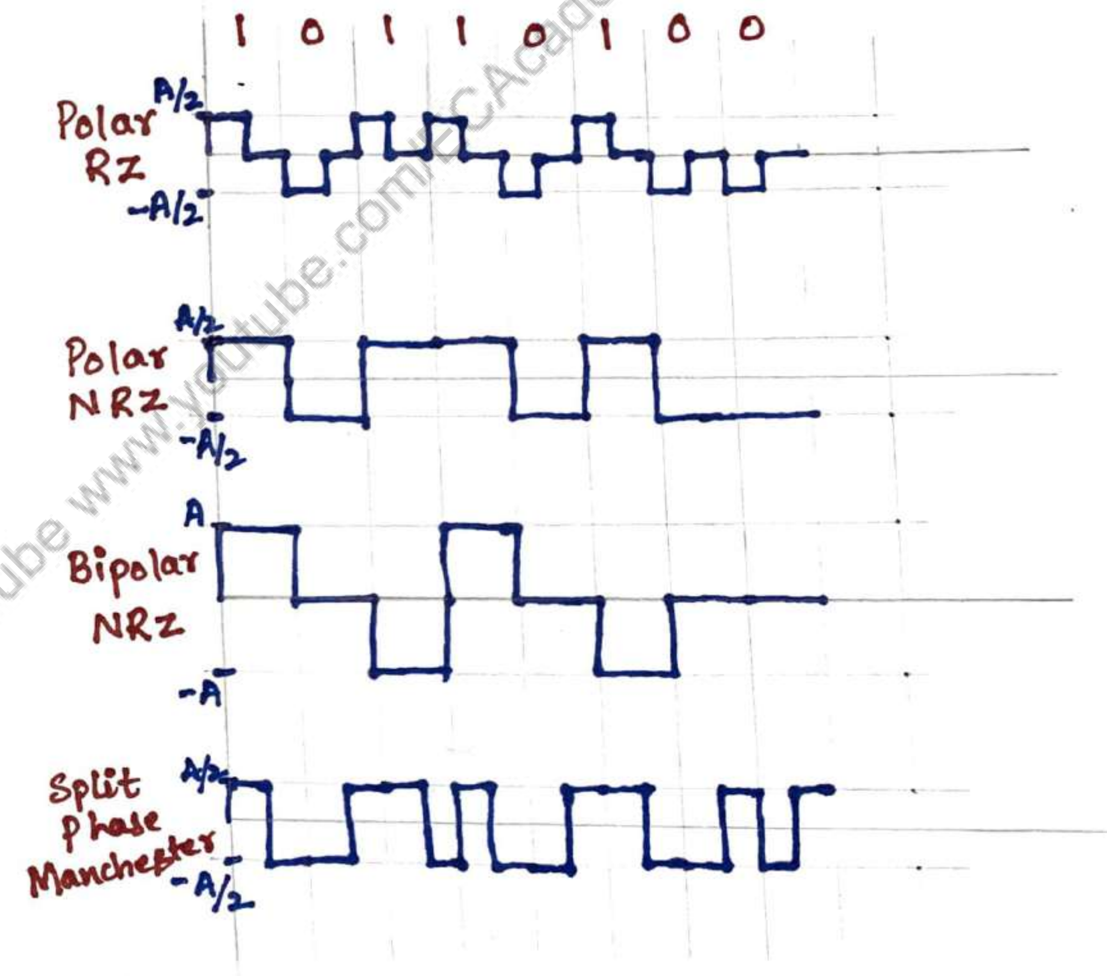
Symbol 1 → $+\frac{A}{2}$; $\frac{T_b}{2}$

0 ; $T_b/2$

Polar NRZ:

Symbol 0 → $-\frac{A}{2}$; T_b

Symbol 1 → $\frac{A}{2}$; T_b



Bipolar NRZ:

Successive 1's → alternative polarities

Symbol '0' → no pulse.

Split phase Manchester:

Symbol 1 → $\frac{A}{2}$; $T_b/2$

$-\frac{A}{2}$; $T_b/2$

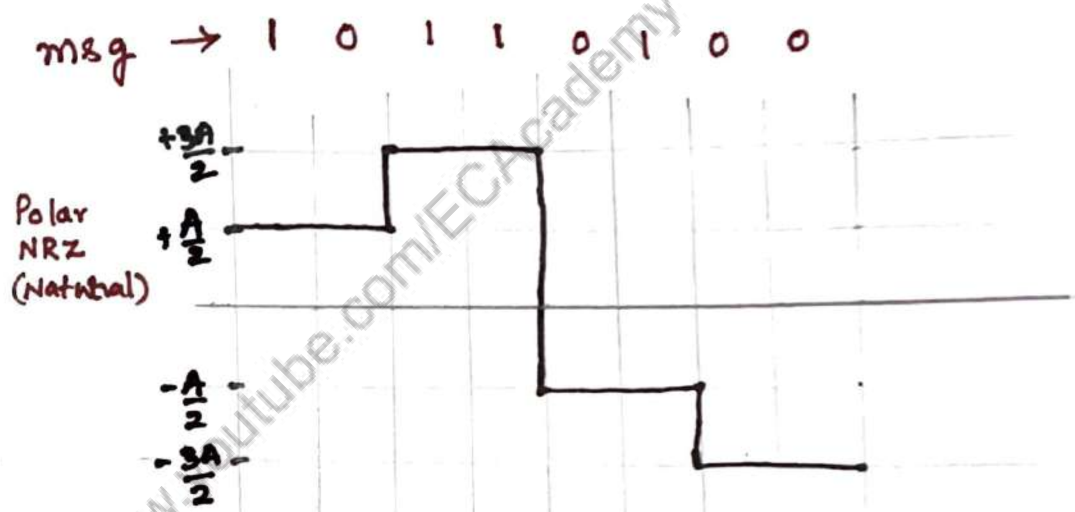
Symbol 0 → $-\frac{A}{2}$; $T_b/2$

$\frac{A}{2}$; $T_b/2$

Polar Quaternary NRZ (Natural code)

msg → grouped in block of 2 bits.

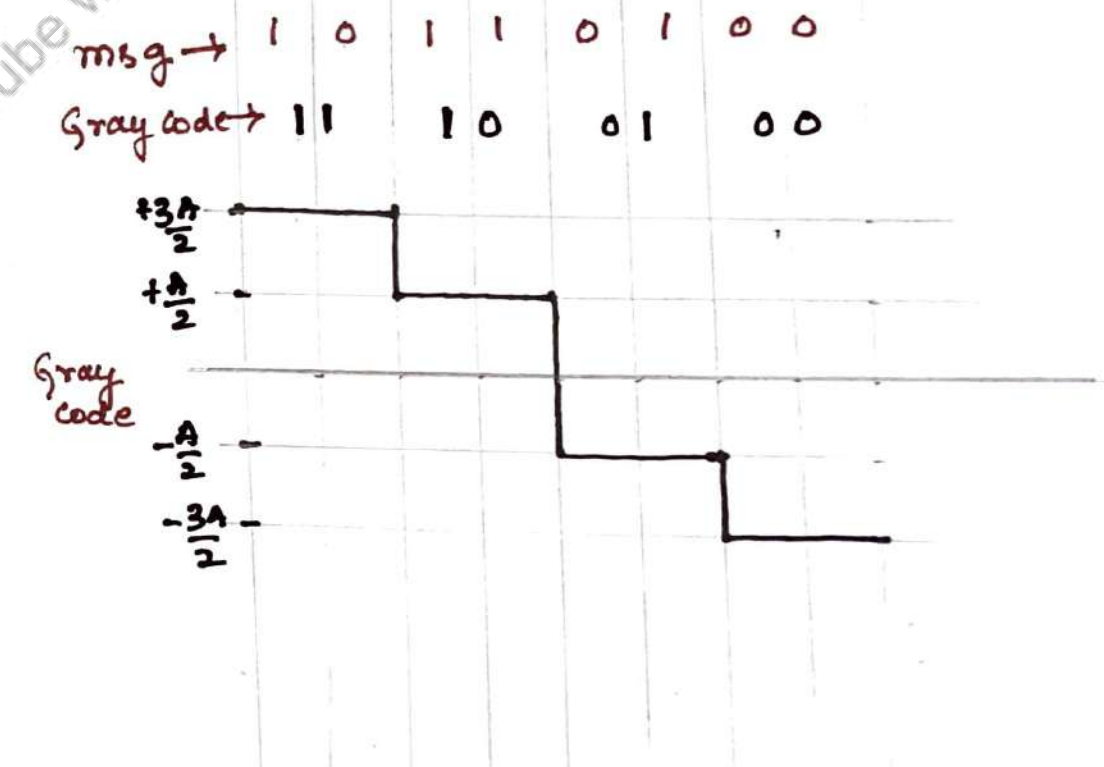
- 0 0 → $-3A/2$
- 0 1 → $-A/2$
- 1 0 → $+A/2$
- 1 1 → $3A/2$



Gray Code:

$g_1 = b_1$ & $g_0 = b_1 \oplus b_0$

msg b_1	msg b_0	Gray code g_1	Gray code g_0	
0	0	0	0	→ $-3A/2$
0	1	0	1	→ $-A/2$
1	0	1	1	→ $+A/2$
1	1	1	0	→ $+3A/2$



M-ary Line code :

K successive bits are grouped

$\therefore$ Symbol $\rightarrow$ $\boxed{M = 2^K}$

Ex:- $K = 3$

$M = 2^3 = 8$ symbols

$000 \rightarrow -7A/8$

$001 \rightarrow -5A/8$

$010 \rightarrow -3A/8$

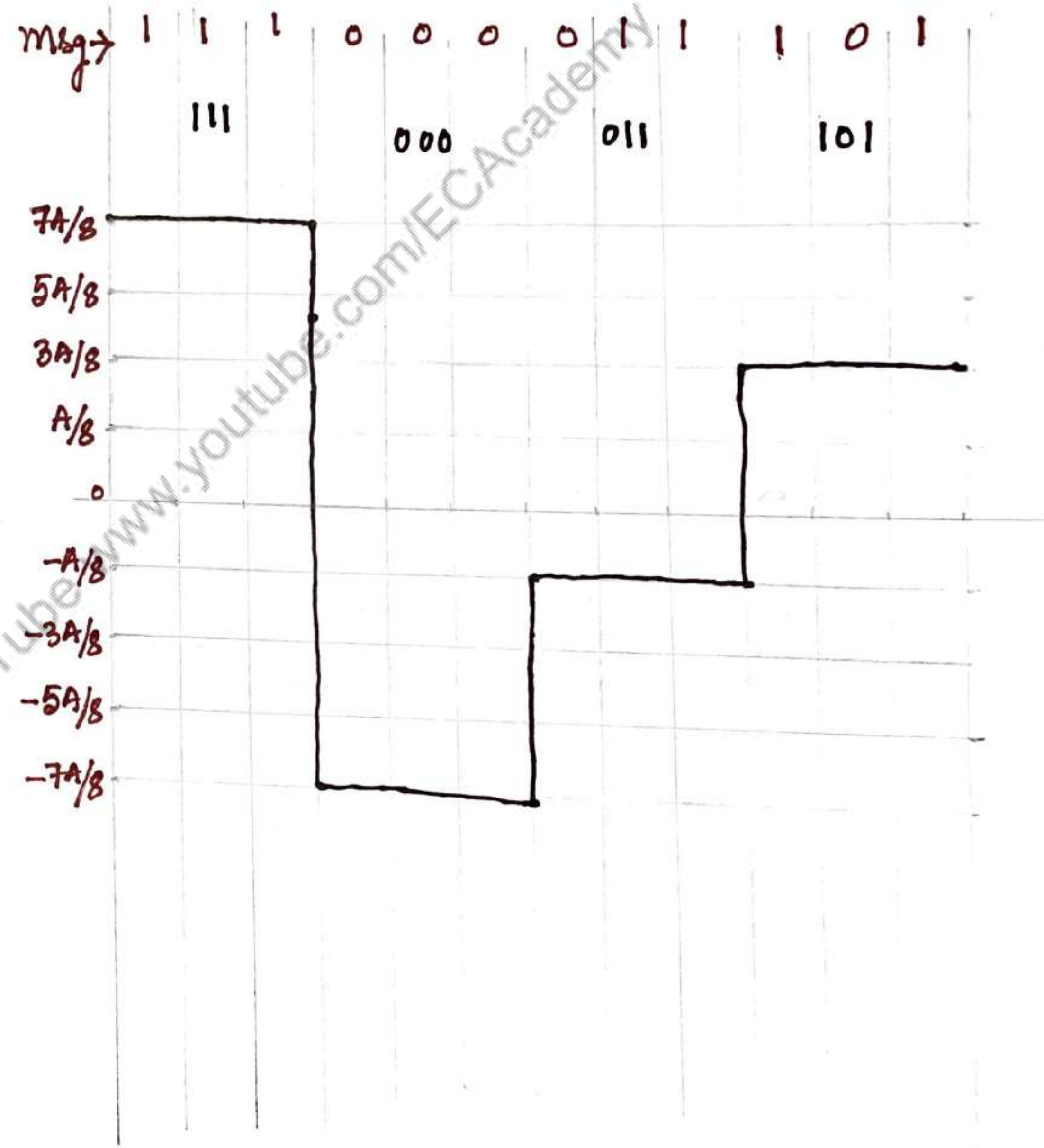
$011 \rightarrow -A/8$

$100 \rightarrow A/8$

$101 \rightarrow 3A/8$

$110 \rightarrow 5A/8$

$111 \rightarrow 7A/8$



M-ary Line code :

K successive bits are grouped

$\therefore$ Symbol $\rightarrow$ $\boxed{M = 2^K}$

Ex:- $K = 3$

$M = 2^3 = 8$ symbols

$000 \rightarrow -7A/8$

$001 \rightarrow -5A/8$

$010 \rightarrow -3A/8$

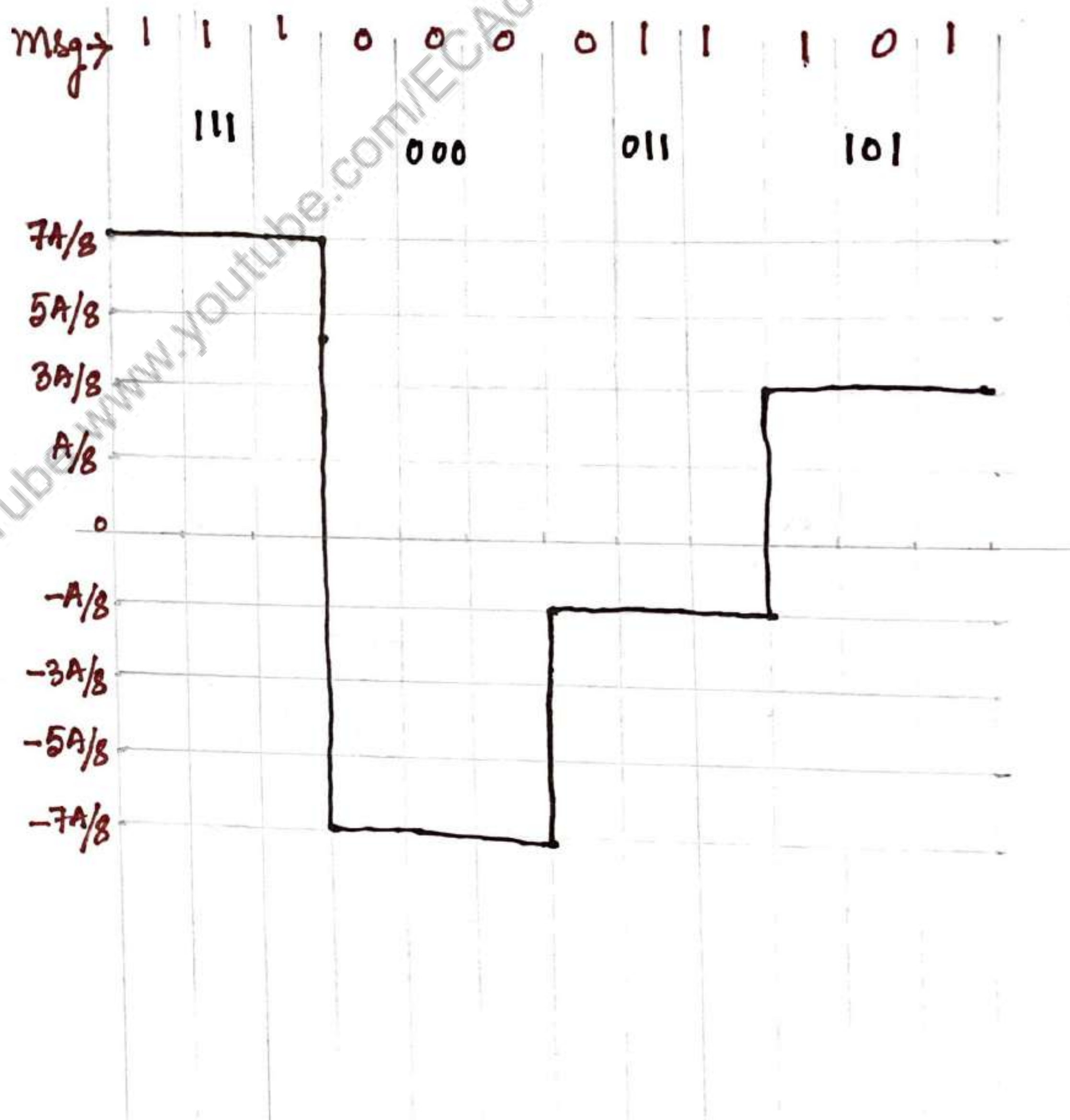
$011 \rightarrow -A/8$

$100 \rightarrow A/8$

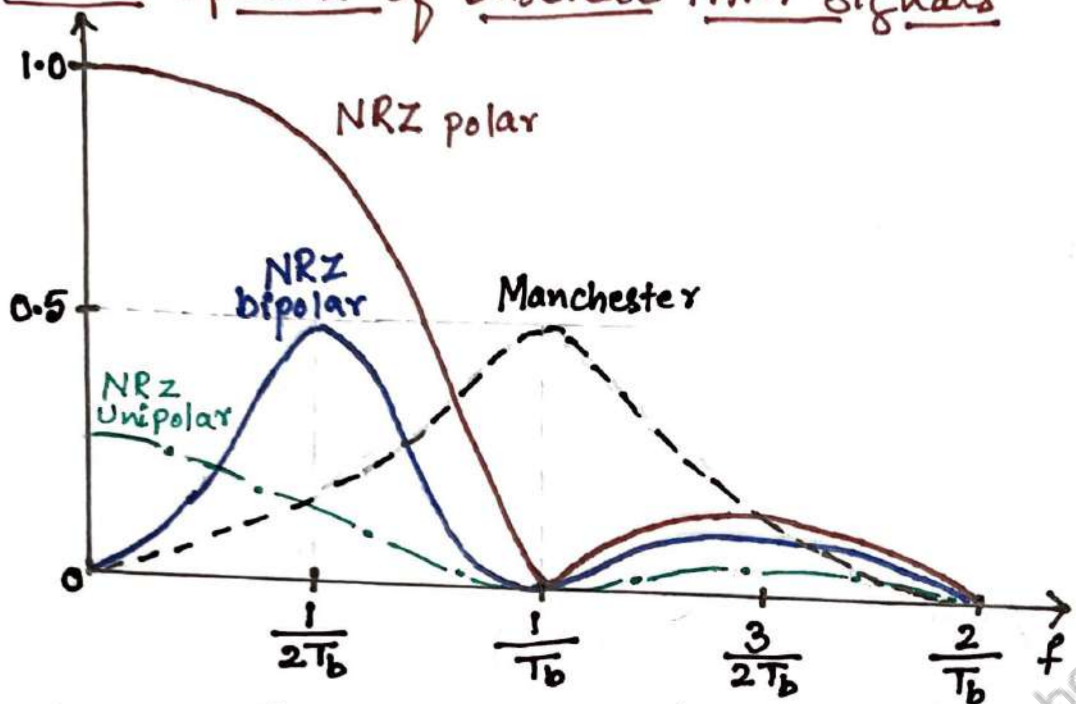
$101 \rightarrow 3A/8$

$110 \rightarrow 5A/8$

$111 \rightarrow 7A/8$



Power Spectra of Discrete PAM Signals



Power Spectra of Various PAM Signals

NRZ Unipolar Format

→ Unipolar → +A → Symbol '0'

→ Signal has some DC component

→ Power lies b/w $\left(\frac{1}{T_b}\right)$ & DC

→ power content above $\frac{1}{T_b}$ is Very small.

NRZ Polar Format

→ Waveform → both +ve & -ve Amplitude

→ Hence → Some DC Value

→ Wave form → Simillar to Sinc pulse

→ power lies b/w DC & $\left(\frac{1}{T_b}\right)$

→ power above $\frac{1}{T_b}$ is Very small.

NRZ bipolar Format

→ successive 1's → pulses of alternating amplitude.

→ Hence no DC component.

→ pulse peak → near $\frac{1}{2T_b}$

→ Power lies inside $\left(\frac{1}{T_b}\right)$

→ Power above $\frac{1}{T_b}$ is Very small.

Manchester Format

→ Every Symbol → +ve & -ve Amplitude

→ Hence no DC component.

→ power lies inside $\left(\frac{2}{T_b}\right)$

→ Width of main pulse → twice of Other format.

→ negligeble power inside $\left(\frac{2}{T_b}\right)$.

HDBN | HDB3 line code:

"High Density Bipolar signalling".

→ Bipolar NRZ → symbol '0' → '0' or No signal.
→ long seq. of zeros.
→ Problem in synchronization.

→ problem is eleminated by adding "PULSES" when no of '0' exceeds 'n' ⇒ $\boxed{\text{HDBN}}$ N=1, 2, 3......

→ If N=3 ⇒ $\boxed{\text{HDB3}}$

→ $\boxed{\text{000V \& B00V}}$ → special sequence.

→ B & V are 1
B → Bipolar rule
V → Violate Bipolar rule.

→ B00V → even no 1's are following special sequence.

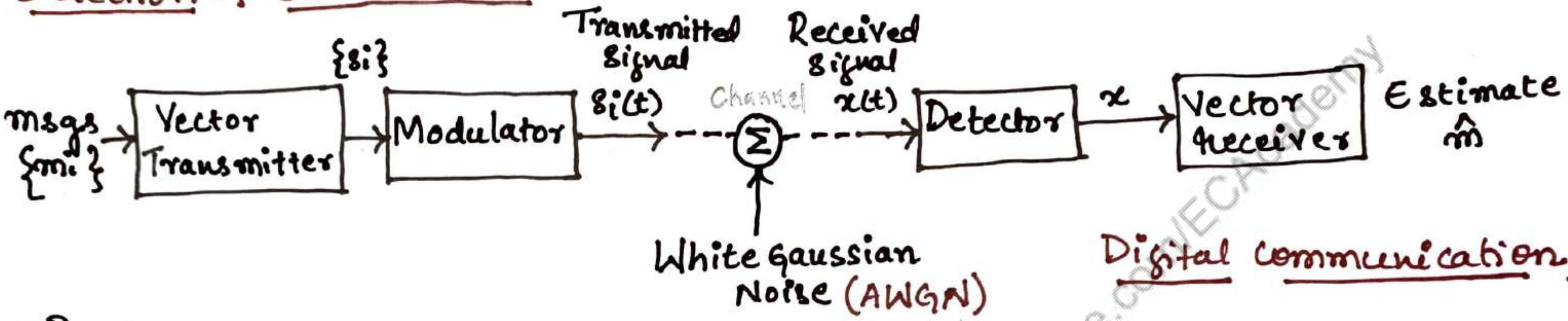
→ 000V → odd no 1's are following special sequence.

input 0 1 0 0 1 1 $\boxed{0\,0\,0\,0}$ 1 0 1 1 0 1 $\boxed{0\,0\,0\,0}$ $\boxed{0\,0\,0\,0}$ 0 0 1 0 1 1 0 1 0 1 $\boxed{0\,0\,0\,0}$ 1

code 0 1 0 0 1 1 0 0 0 V 1 0 1 1 0 1 B 0 0 V B 0 0 V 0 0 1 0 1 1 0 1 0 1 0 0 0 V 1

Bipolar NRZ

Detection & Estimation:



Digital communication s/m

→ Receiver observes the received signal & determines which symbol was transmitted.
⇒ Detection

→ Receiver can use the information of received signal to extract the estimates of physical parameter ⇒ Estimation

→ msgs $\{m_i\} = m_1, m_2, m_3 \cdots m_M$

Probability $P(m_i) = \frac{1}{M}$

→ Vector $s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ s_{i3} \\ \vdots \\ s_{iN} \end{bmatrix}$ $\quad i = 1, 2, 3 \ldots M$

here $N \leq M$

→ Modulator constructs the waveform $s_i(t)$ from the symbol s_i

$$E_i = \int_0^T s_i^2(t)\,dt \;;\; i = 1, 2, \ldots M.$$

→ Signal is transmitted over a noisy channel.

$$\therefore X(t) = s_i(t) + N(t)$$

$X(t)$ → Received random process.

$x(t)$ → Sample of $X(t)$ → received signal

→ Detector ⟶ Processes $x(t)$ & Produce x

→ Vector receiver → Obtain $\hat{m}$

→ Error ⇒ $\hat{m} \neq m_i$ $\qquad P_e = P(\hat{m} \neq m_i)$

Geometric Representation of signal

Consider 'M' no of energy signals,

$$s_i(t) = \{ s_1(t), s_2(t), s_3(t) \text{ ----- } s_M(t) \}$$

in terms of 'N' no of orthornal basis fun

$$\therefore \phi_j(t) = \{ \phi_1(t), \phi_2(t) \text{ ---- } \phi_N(t) \}$$

linear relation ship b/w $s_i(t)$ & $\phi_j(t)$ as,

$$s_i(t) = s_{i1}\phi_1 + s_{i2}\phi_2 + \text{ ----- } + s_{iM}\phi_N(t)$$

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t) \longrightarrow (1)$$

$s_{ij} \rightarrow$ Coefficients of expantion.

$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt \longrightarrow (2)$$

$T \rightarrow$ the duration of symbol $s_i(t)$

$\phi_1(t), \phi_2(t)$ ---- $\phi_N(t)$ are orthonormal basis

$$\int_0^T \phi_i(t)\phi_j(t)\,dt = \begin{cases} 1 ; & i = j \quad \text{energy} \\ 0 ; & i \neq j \quad \text{Orthogonal} \end{cases}$$

Ex:- Twodiamentional signal with three symbol $M=3$ $s_1\ s_2\ s_3$

$N=2$ $\phi_1\ \phi_2$

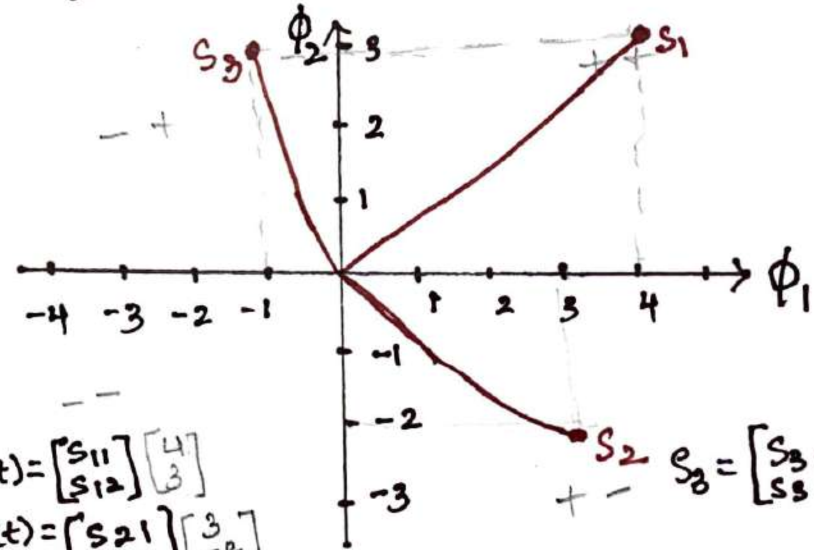
$$s_i(t) = \sum_{j=1}^{2} s_{ij}(t)\,\phi_j(t) \ ; \ i = 1,2,3.$$

$$s_i(t) = s_{i1}(t)\phi_1(t) + s_{i2}(t)\phi_2(t)$$

$$s_1(t) = s_{11}(t)\phi_1(t) + s_{12}(t)\phi_2(t)$$

$$s_2(t) = s_{21}(t)\phi_1(t) + s_{22}(t)\phi_2(t)$$

$$s_3(t) = s_{31}(t)\phi_1(t) + s_{32}(t)\phi_2(t)$$



$s_1(t) = \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$s_2(t) = \begin{bmatrix} s_{21} \\ s_{22} \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$s_3 = \begin{bmatrix} s_{31} \\ s_{32} \end{bmatrix} \begin{matrix} -1 \\ 3 \end{matrix}$

ϕ_1 & $\phi_2 \rightarrow$ perpendicular to each other

$\phi_1 - \phi_2 \rightarrow$ Euclidean Space

fig $\rightarrow$ 2D Euclidean space

Relationship between Signal Energy & it's Vector

WKT energy of a signal $s_i(t)$

$$E_i = \int_0^T s_i^2(t)\,dt \qquad \because s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t)$$

$$\therefore E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\,\phi_j(t)\right]\left[\sum_{K=1}^{N} s_{iK}\,\phi_K(t)\right] dt$$

Rearrange,

$$E_i = \sum_{j=1}^{N}\sum_{K=1}^{N} s_{ij}\cdot s_{iK}\int_0^T \phi_j(t)\,\phi_K(t)\,d(t) \longrightarrow (1)$$

$$\because \int_0^T \phi_j(t)\,\phi_K(t) = \begin{cases} 1 & ; j=K \\ 0 & ; j\neq K \end{cases}$$

$$E_i = \begin{cases} \sum_{j=1}^{N}\sum_{K=1}^{N} s_{ij}\,s_{iK} & ; j=K \\ 0 & ; j\neq K \end{cases}$$

$$\therefore E_i = \sum_{j=1}^{N}\sum_{j=1}^{N} s_{ij}\,s_{ij} \quad ; j=K$$

$$E_i = \sum_{j=1}^{N} s_{ij}^2 \longrightarrow (2)$$

$$\because \sum_{j=1}^{N} s_{ij}^2 = \underbrace{\|s_i\|^2}_{\text{Length of signal vector}}$$

$$\therefore \boxed{E_i = \|s_i\|^2}$$

$$\because \|s_i\|^2 = s_i^T\cdot s_i$$

$$\boxed{E_i = s_i^T\cdot s_i}$$

Euclidean distance:

$$d_{iK} = \| s_i - s_K \| \longrightarrow (1)$$

$$\because \| s_i - s_K \|^2 = \sum_{j=1}^{N} (s_{ij} - s_{Kj})^2 \longrightarrow (2)$$

$$\text{if } E_i = \int_0^T s_i^2(t)\,dt = \| s_i \|^2$$

$$\therefore (2) \Rightarrow \| s_i - s_K \|^2 = \int_0^T [s_i(t) - s_K(t)]^2 dt \longrightarrow (3)$$

The angle b/w the two vectors s_i & s_K,

$$\cos \theta_{iK} = \frac{s_i^T \cdot s_K}{\| s_i \| \, \| s_K \|}$$

$s_i^T \cdot s_K = 0$ $\therefore$ two vectors are said to be orthogonal.

$$\therefore \boxed{\theta_{iK} = 90^\circ}$$

Gram Schmidt Orthogonalization Procedure:

M energy signal $\rightarrow$ $S_1(t), S_2(t) \cdots S_M(t)$

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}} \longrightarrow (1)$$

$E_1 \rightarrow$ energy of signal $S_1(t)$

$$S_1(t) = \sqrt{E_1}\,\underbrace{\phi_1(t)}_{\text{unity energy}} = S_{11}(t)\,\phi_1(t)$$

$$S_{21} = \int_0^T S_2(t)\,\phi_1(t)\,dt$$

$$\underbrace{g_2(t)}_{\rightarrow \text{ new intermidiate fun}} = S_2(t) - S_{21}\,\phi_1(t) \longrightarrow (2)$$

$g_2(t)$ is orthogonal to $\phi_1(t)$; $0 \le t \le T$

$$\underbrace{\phi_2(t)}_{\rightarrow \text{ 2nd basis fun}} = g_2(t) \Big/ \sqrt{\int_0^T g_2^2(t)\,dt} \longrightarrow (3)$$

Substitute (2) in (3)

$$\phi_2(t) = \frac{S_2(t) - S_{21}\,\phi_1(t)}{\sqrt{\int_0^T \left[S_2(t) - S_{21}\phi_1(t)\right]^2 dt}}$$

$$\therefore \phi_2(t) = \frac{S_2(t) - S_{21}\,\phi_1(t)}{\sqrt{E_2 - S_{21}^2}}$$

$E_2 \rightarrow$ energy of $S_2(t)$

$$\therefore \int_0^T \phi_2(t)\,dt = 1$$

$$\int_0^T \underbrace{\phi_1(t)\,\phi_2(t)}_{\text{Orthonormal}} = 0$$

Gram Schmidt Orthogonalization Procedure:

Generally $g_i(t) = S_i(t) - \sum_{j=1}^{i-1} S_{ij}\, \phi_j(t)$

$$S_{ij} = \int_0^T S_i(t)\, \phi_j(t)\, dt \;;\; j = 1, 2, \ldots, i-1$$

for $i = 1$; $g_i(t) \longrightarrow S_i(t)$

$\therefore$ for $g_i(t) \longrightarrow$ define the basis fun$^{=}$

$$\boxed{\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}}} \;;\; j = 1, 2, \ldots N$$

$N \leq M$

Conversion of the Continuous AWGN Channel into a Vector Channel:

$$\underbrace{x(t)}_{\text{noisy signal}} = S_i(t) + \underbrace{w(t)}_{\substack{\text{Sample fun} \\ \text{WGN process}}} \longrightarrow (1)$$

O/P of Correlator $\rightarrow$ a random Variable

$$x_j = \int_0^T x(t)\,\phi_j(t)\,dt \longrightarrow (2)$$

$$= \underbrace{S_{ij}}_{\substack{\text{deterministic} \\ \text{component of } x_j \rightarrow S_i(t)}} + W_j \rightarrow \text{Sample Value of the random Variable } W_j$$

$$S_{ij} = \int_0^T S_i(t)\,\phi_j(t)\,dt \longrightarrow (3)$$

$$W_j = \int_0^T w(t)\,\phi_j(t)\,dt \longrightarrow (4)$$

new process $X'(t) \rightarrow$ sample fun $x'(t)$

$$\therefore x'(t) = x(t) - \sum_{j=1}^{N} x_j\,\phi_j(t) \longrightarrow (5)$$

put eqn (1) & eqn (2) in (5)

$$S_i(t) = \sum_{j=1}^{N} S_{ij}\,\phi_j(t)$$

$$x'(t) = S_i(t) + w(t) - \sum_{j=1}^{N} (S_{ij} + w_j)\,\phi_j(t)$$

$$= w(t) - \sum_{j=1}^{N} w_j\,\phi_j(t)$$

$$x'(t) = w'(t)$$

$$x(t) = \sum_{j=1}^{N} x_j\,\phi_j(t) + x'(t)$$

$$x(t) = \sum_{j=1}^{N} x_j\,\phi_j(t) + w'(t) \rightarrow \text{random Process}$$

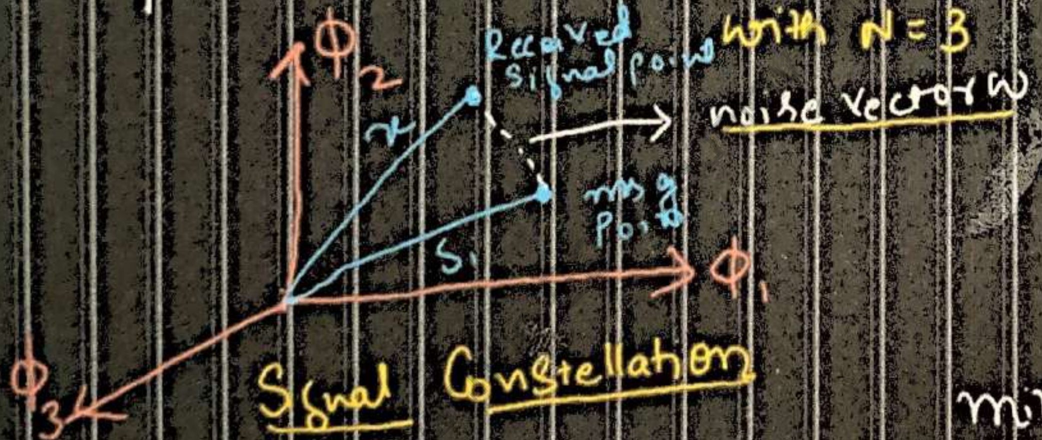
Optimum recivers using Coherent detection / Maximum Likelyhood decoding

transmitted signal $\longrightarrow$ currupted by noise

$$x(t) = S_i(t) + \underbrace{w(t)}_{\substack{\text{Sample fun of} \\ \text{AWGN process } w(t)}} \longrightarrow (1)$$

Reciver $\rightarrow$ observe $x(t)$ & make best estimation of $S_i(t)$

Let us represent S_i, x & w $\Rightarrow$ Euclidean Space with $N = 3$



Signal Constellation

Detector $\rightarrow$ observers the x & perform the mapping estimation $\hat{m}$ for m_i

ML decoding:

Let $x \rightarrow$ observation vector

decision $\longrightarrow$ $\hat{m} = m_i$

avg Probability of Symbol error

$$\underbrace{P_e(m_i, x)}_{\substack{\text{avg Probability} \\ \text{of Symbol error}}} = 1 - P(m_i \text{ sent} / x) \rightarrow (2)$$

min error $\Rightarrow$ optimum decision rule

ML decoding.

optimum decision rule

Set $\hat{m} = m_i$ if $P(m_i \text{ Sent}/x) \geq P(m_k \text{ Sent}/x)$

for all $k \neq i$

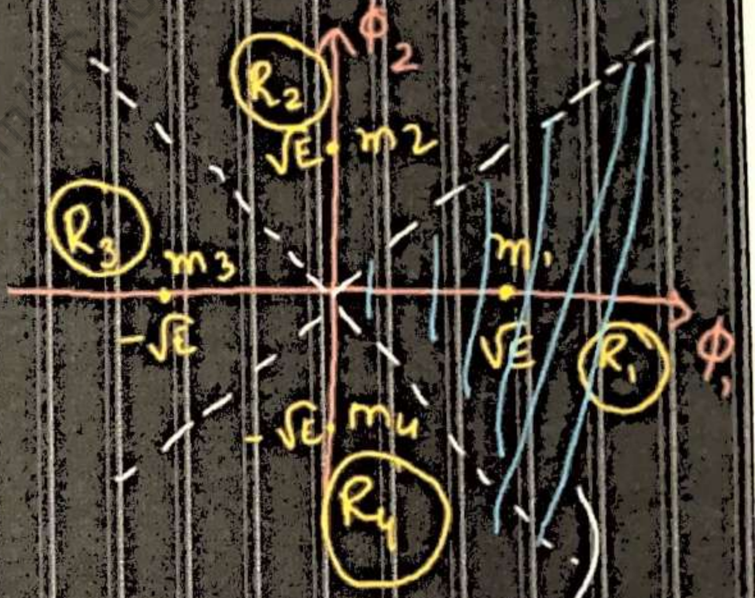
graphically:

$R \rightarrow$ N-diamentional space of all possible vector x

Partioned $\rightarrow$ M decision regions $R_1, R_2, \ldots R_M$

decision rule, $\|x - S_k\|$ is min for $k = i$

Ex: $N = 2$ $M = 4 \longrightarrow R = 4$

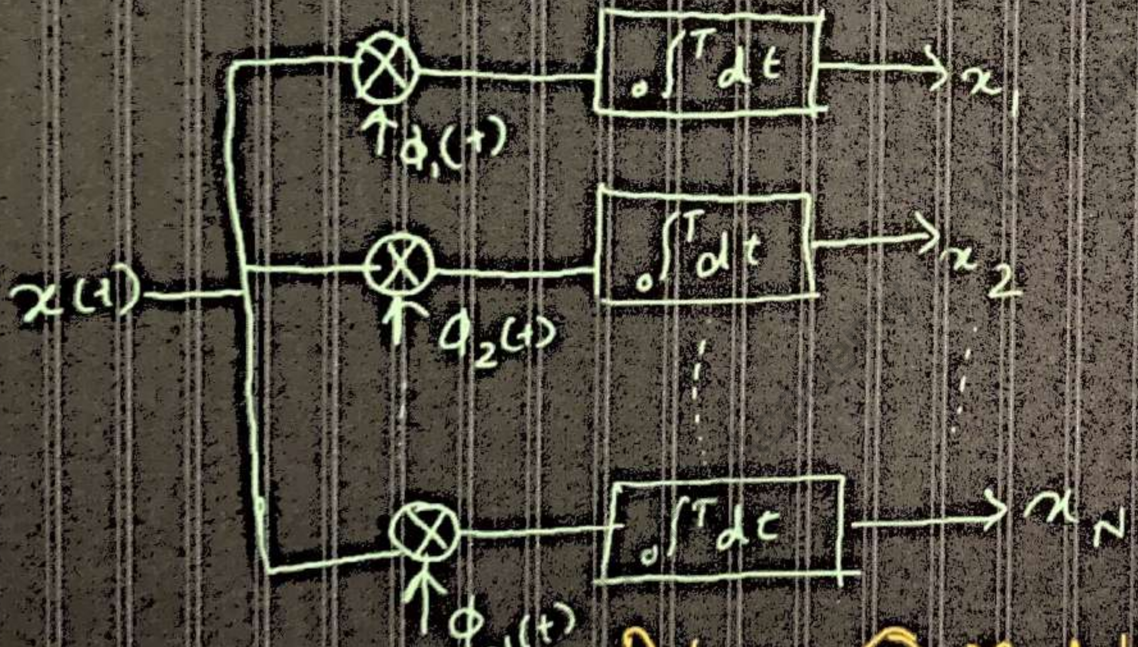


Select m_1 if x lies in this region.

Correlation Receivers:

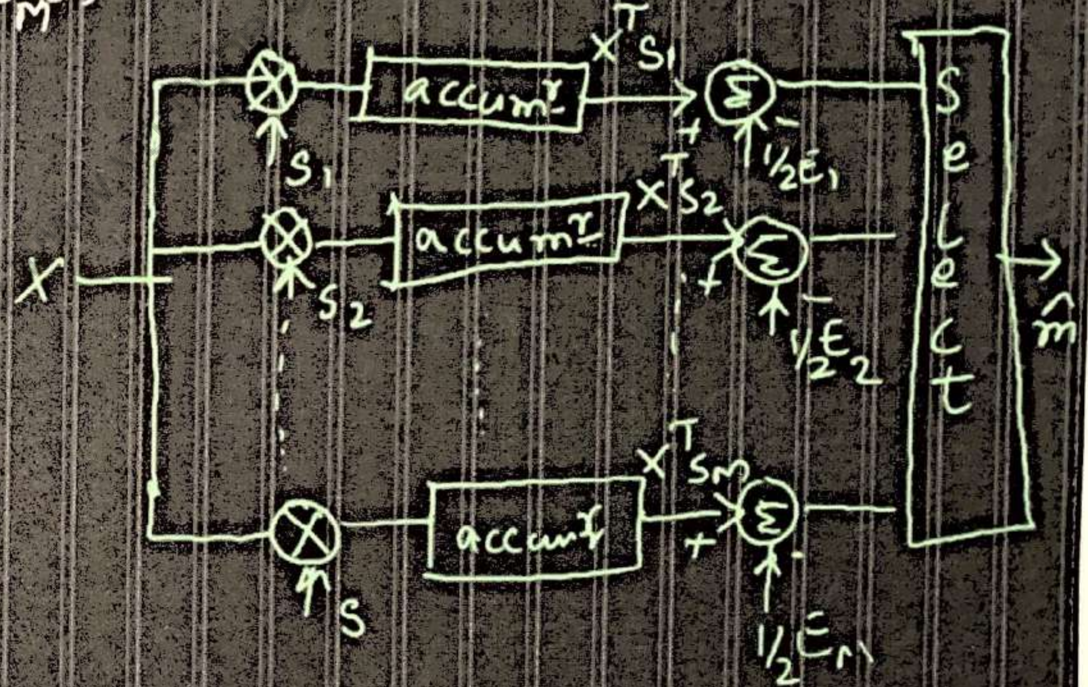
Optimum receivers for an AWGN channel when the transmitted signal $s_1(t), s_2(t) \dots s_M(t)$ are even likely

① **Detectector:**



Detector (or) Modulator

② **ML decoder:**



Matched Filter Receiver:

Correlator → different equivalent structure

LTI filter → $h_j(t)$ & $\underbrace{x(t)}_{i/p}$

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, h_j(t-\tau)\, d\tau$$

$$y_j(T) = \int_{-\infty}^{\infty} x(\tau)\, h_j(T-\tau)\, dt \longrightarrow (1)$$

$0 \le t \le T$

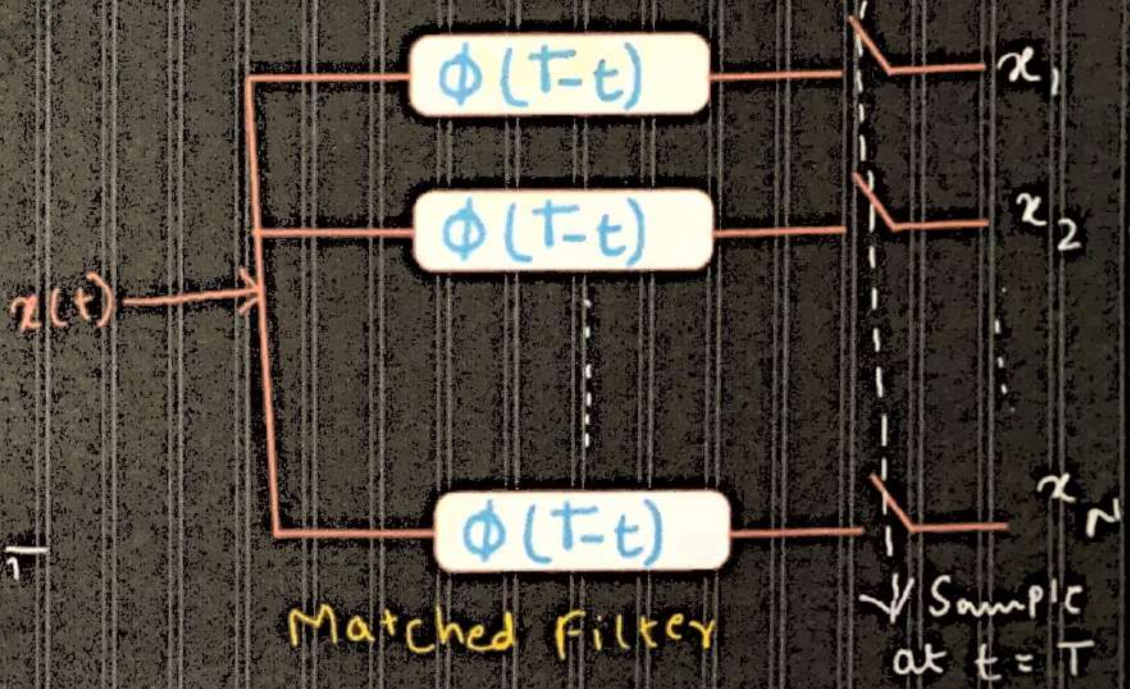
next detector → o/p j^{th} correlator

$$x_j = \int_0^T x(t)\, \phi_j(t)\, dt \longrightarrow (2)$$

$h_j(T-t) = \phi_j(t)$; $0 \le t \le T$

||ly $h_j(t) = \phi_j(T-t)$, $0 \le t \le T$

$$\boxed{h(t) = \phi(T-t) \ ; \ 0 \le t \le T}$$



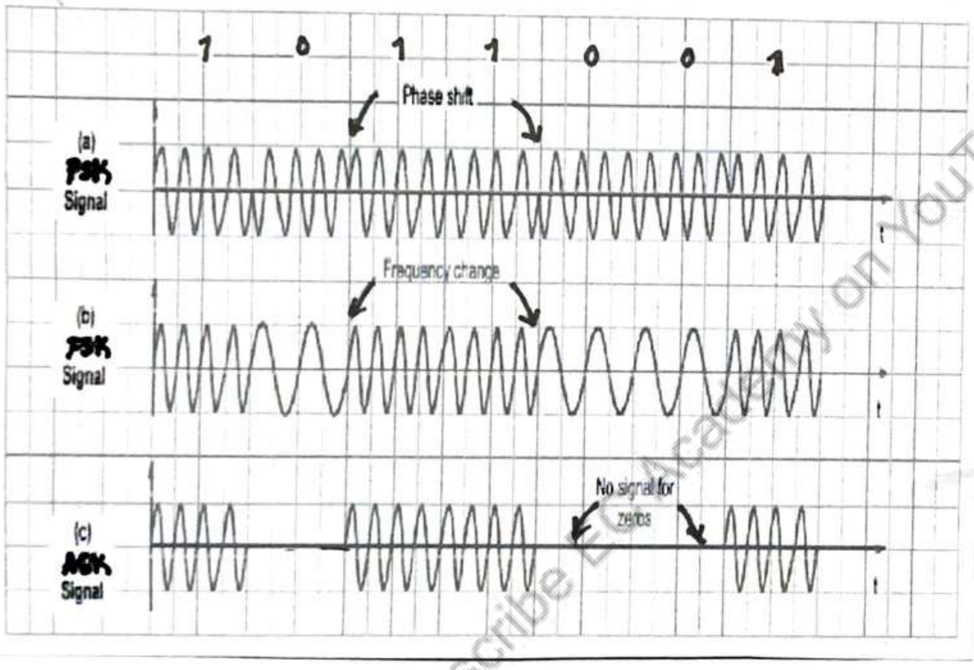
Digital Modulation Techniques:

Digital Commn

1. Base band transmission -
2. Pass band transmission. -

Digital Modulation

technique that uses discrete Signals to modulate a Carrier wave



① Phase shift Keying (PSK)
Digital data modulates the phase of the carrier signal

② Frq Shift Keying (FSK)
Digital data modulates the Frq of the Carrier Signal.

③ Amplitude Shift Keying (ASK)
Digital data modulates the Amplitude of the Carrier Signal.

Type of Reception of Data. [Pass band]

① Coherent (Synchronous)
Carrier at receiver & transmitter → "Phase Locked"

② Non Coherent (Envelope)
Carrier at receiver & transmitter → "Not Phase locked"

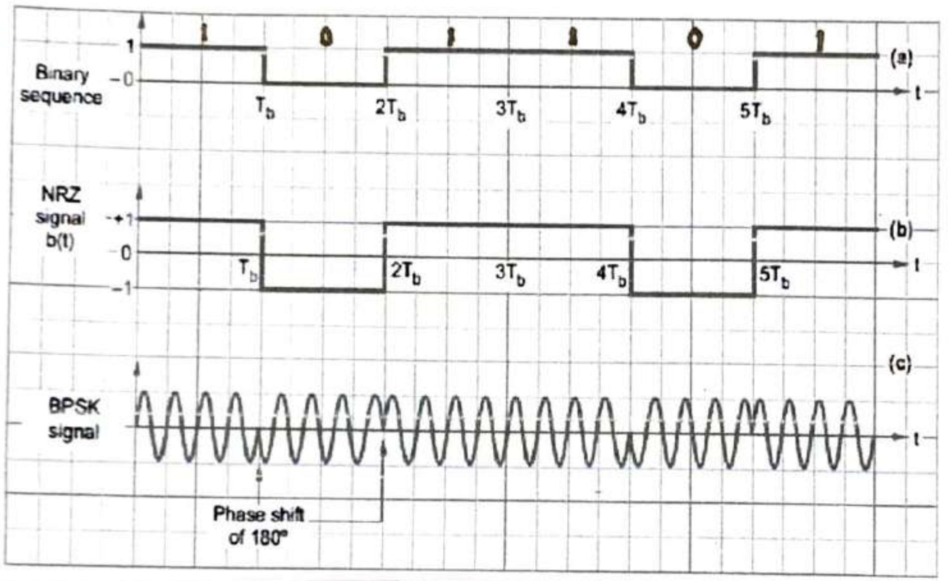
Advantages

→ Long distance transmission
→ No Crosstalk
→ Analog channels can be used
→ Wireless channels can be used
→ Many Modulation techniques.

Disadvantages

→ S/m is Complex
→ not suitable for short distance commn.

Binary Phase Shift Keying [BPSK] using coherent detection:



→ BPSK ⇒ Binary symbols '0 & 1'

Carrier signal

$$s(t) = A\cos(2\pi f_0 t)$$

A → Peak value of sinusoidal carrier signal.

Power dissipated

$$P = \frac{1}{2}A^2 \Rightarrow A = \sqrt{2P}$$

Symbol 1:

$$s_1(t) = \sqrt{2P}\cos(2\pi f_0 t) \longrightarrow (1)$$

If next symbol is zero

Symbol 0:

There will be a phase shift of 180° ($\pi \rightarrow$ radians)

$$s_2(t) = \sqrt{2P}\cos(2\pi f_0 t + \pi)$$

$$\because \cos(\theta + \pi) = -\cos\theta$$

$$s_2(t) = -\sqrt{2P}\cos(2\pi f_0 t) \longrightarrow (2)$$

using eqn (1) & (2)

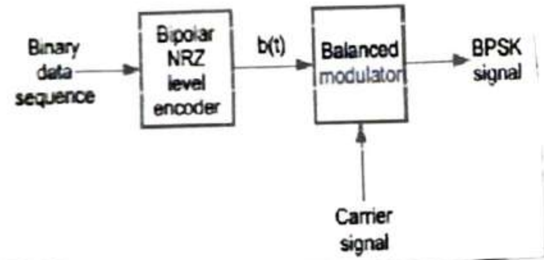
$$\boxed{s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)}$$

$\boxed{b(t) = +1} \Rightarrow$ symbol '1'

$\boxed{b(t) = -1} \Rightarrow$ symbol '0'

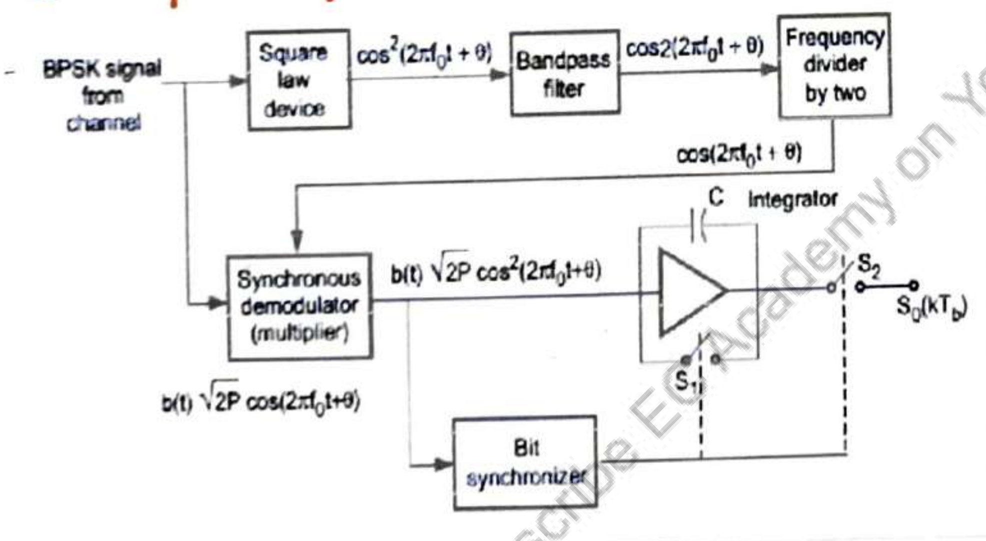
Generation and Reception of BPSK signal.

(a) Generation of BPSK signal: [Transmitter]



$$S(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t) \longrightarrow (1)$$

(b) Reception of BPSK signal: [Receiver]



Phase change $[\theta] \Rightarrow S(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta) \longrightarrow (2)$

(i) Square law device: $\cos^2(2\pi f_0 t+\theta)$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \cos^2(2\pi f_0 t+\theta) = \frac{1+\cos 2(2\pi f_0 t+\theta)}{2}$$

$$\Rightarrow \underbrace{\frac{1}{2}}_{\text{DC level.}} + \frac{1}{2}\cos 2(2\pi f_0 t+\theta)$$

(ii) Band pass filter: $\cos 2(2\pi f_0 t+\theta)$

(iii) Frq. devider: $\cos(2\pi f_0 t+\theta)$

(iv) Synchronous demodulator

$$b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta) \times \cos(2\pi f_0 t+\theta)$$

$$\Rightarrow b(t)\sqrt{2P}\cos^2(2\pi f_0 t+\theta)$$

$$\Rightarrow b(t)\sqrt{2P}\left\{\frac{1}{2}\left[1+\cos 2(2\pi f_0 t+\theta)\right]\right\}$$

$$\Rightarrow b(t)\sqrt{\frac{P}{2}}\left\{1+\cos 2(2\pi f_0 t+\theta)\right\}$$

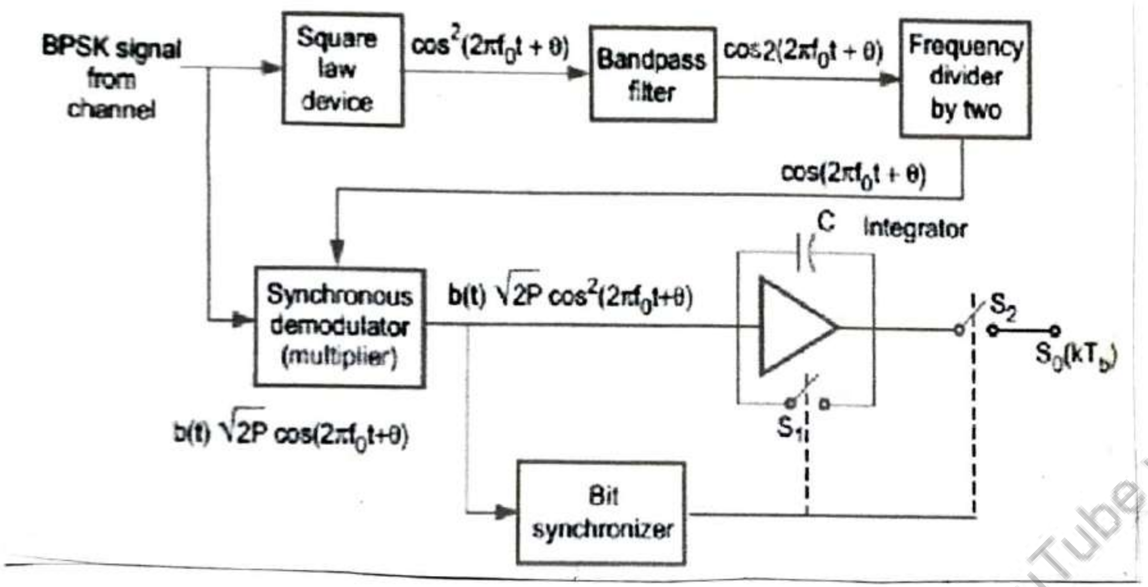
(Vi) Bit synchronizer and Integrator:

Bit synchronizer ↳ take care of starting & end time of a bit.

Integrator ↓ Integrate the signal over one bit period.

Reception of BPSK Signal:

Output of integrator depends on transmitted bit:



$$s(t) = b(t)\sqrt{2P}\cos^2(2\pi f_0 t+\theta)$$

$$s(t) = b(t)\sqrt{\frac{P}{2}}\left[1+\cos 2[2\pi f_0 t+\theta]\right]$$

K^{th} bit interval

$$s_0(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\int_{(K-1)T_b}^{KT_b}\left[1+\cos 2[2\pi f_0 t+\theta]\right]dt$$

$T_b \rightarrow$ one bit period.

$$s_0(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\left\{\int_{(K-1)T_b}^{KT_b} 1\,dt + \cancel{\int_{(K-1)T_b}^{KT_b}\cos 2[2\pi f_0 t+\theta]\,dt}^{\,0}\right\}$$

Avg value of sin wave = 0 if integration for full cycle

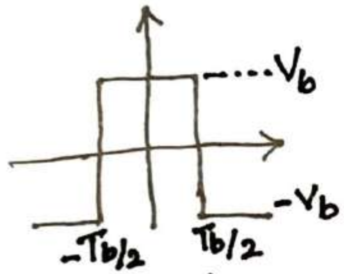
$$s_0(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\left[t\right]_{(K-1)T_b}^{KT_b}$$

$$s_0(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\left\{KT_b-(K-1)T_b\right\}$$

$$\cancel{KT_b} - \cancel{KT_b} + T_b$$

$$\boxed{s_0(KT_b) = b(KT_b)\sqrt{\frac{P}{2}}\,T_b}$$

Spectrum of BPSK Signal:



NRZ pulse

Fourier transform

$$X(f) = V_b T_b \frac{\sin(\pi f T_b)}{\pi f T_b} \rightarrow ①$$

PSD of NRZ pulse as $S(f) = \frac{\overline{X(f)^2}}{T_s}$

$\overline{X(f)} \rightarrow$ avg value of $X(f)$

$T_s \rightarrow$ symbol duration.

$$S(f) = \frac{V_b^2 T_b^2}{T_s} \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

BPSK signal $\rightarrow$ one bit is transmitted at a time

$\therefore T_s = T_b$

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2 \rightarrow ②$$

$\pm\sqrt{P}$ $\qquad$ sinc fun

PSD of BPSK Signal.

BPSK signal $\Rightarrow$ Modulating carrier by NRZ pulse $(b(t))$

$f_0 \rightarrow$ carrier freq. The spectral components $\Rightarrow$ f_0+f, f_0-f

$$S_{BPSK}(f) = V_b^2 T_b \left\{\frac{1}{2}\left[\frac{\sin(\pi(f_0-f)T_b)}{\pi(f_0-f)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin(\pi(f_0+f)T_b)}{\pi(f_0+f)T_b}\right]^2\right\}$$

$V_b = \pm\sqrt{P}$

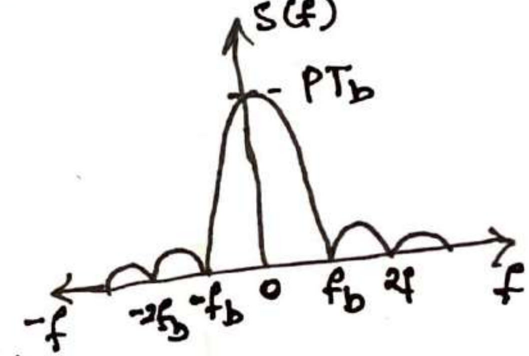
$$\boxed{S_{BPSK}(f) = \frac{PT_b}{2}\left\{\left[\frac{\sin(\pi(f_0-f)T_b)}{\pi(f_0-f)T_b}\right]^2 + \left[\frac{\sin\pi(f_0+f)T_b}{\pi(f_0+f)T_b}\right]^2\right\}} \rightarrow ③$$

Modulating signal, $s(t) = \pm\sqrt{2P}\cos(2\pi f_0 t)$

if $b(t) = \pm\sqrt{P}$ then, $s(t) = \sqrt{2}\cos(2\pi f_0 t)$

Plot of PSD

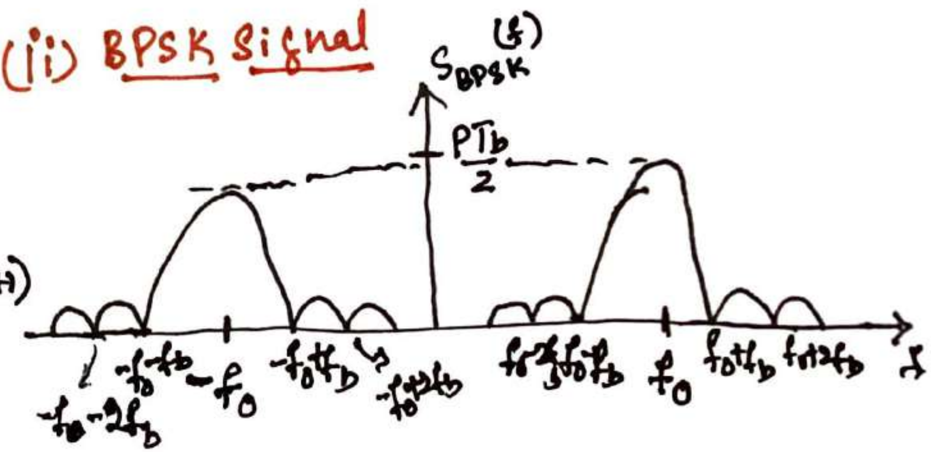
(i) NRZ pulse



$f_b = \frac{1}{T_b}$

$PT_b \rightarrow$ amplitude

(ii) BPSK Signal



Geometrical Representation of BPSK Signals:

BPSK Signal → two symbols → 1, 0

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t) \rightarrow ①$$

× & ÷ eqn① by $\sqrt{PT_b}$

$$\left\{ \frac{\sqrt{2P}}{\sqrt{PT_b}} = \sqrt{\frac{2}{T_b}} \right.$$

$$① \Rightarrow s(t) = b(t)\sqrt{PT_b}\cdot\underbrace{\sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t)}_{\phi_1(t)} \rightarrow ②$$

$\phi_1(t)$ ↳ Carrier Signal

$$② \Rightarrow s(t) = b(t)\sqrt{PT_b}\,\phi_1(t) \rightarrow ③$$

bit energy 'E_b' as $\boxed{E_b = PT_b}$

$P \rightarrow$ Power

$T_b \rightarrow$ bit duration.

$$③ \Rightarrow \boxed{s(t) = b(t)\sqrt{E_b}\,\phi_1(t)} \rightarrow ④$$

$b(t) = +1 \rightarrow$ Symbol '1'

$b(t) = -1 \rightarrow$ Symbol '0'

$s(t) = \sqrt{E_b}\,\phi_1(t) \rightarrow$ Symbol '1'

$s(t) = -\sqrt{E_b}\,\phi_1(t) \rightarrow$ Symbol '0'

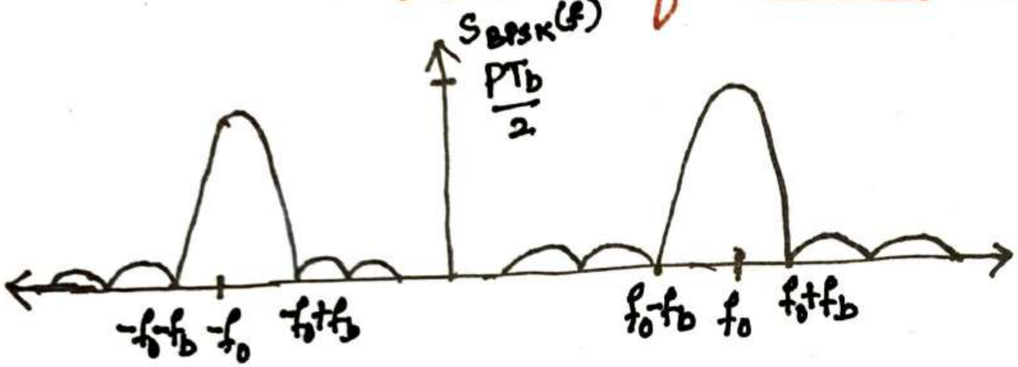
$$d = \sqrt{E_b} - (-\sqrt{E_b})$$

$$\boxed{d = 2\sqrt{E_b}}$$

as distance increases the isolation increases.

∴ the probability of error reduces.

Band width of BPSK Signal:



$T_b \rightarrow$ bit duration

$\therefore f_b = \frac{1}{T_b}$

$\hookrightarrow$ max frq in base band signal.

$\therefore$ BW = Highest frq - lowest frq in main lobe

$$BW = f_0 + f_b - (f_0 - f_b)$$

$$BW = \cancel{f_0} + f_b - \cancel{f_0} + f_b$$

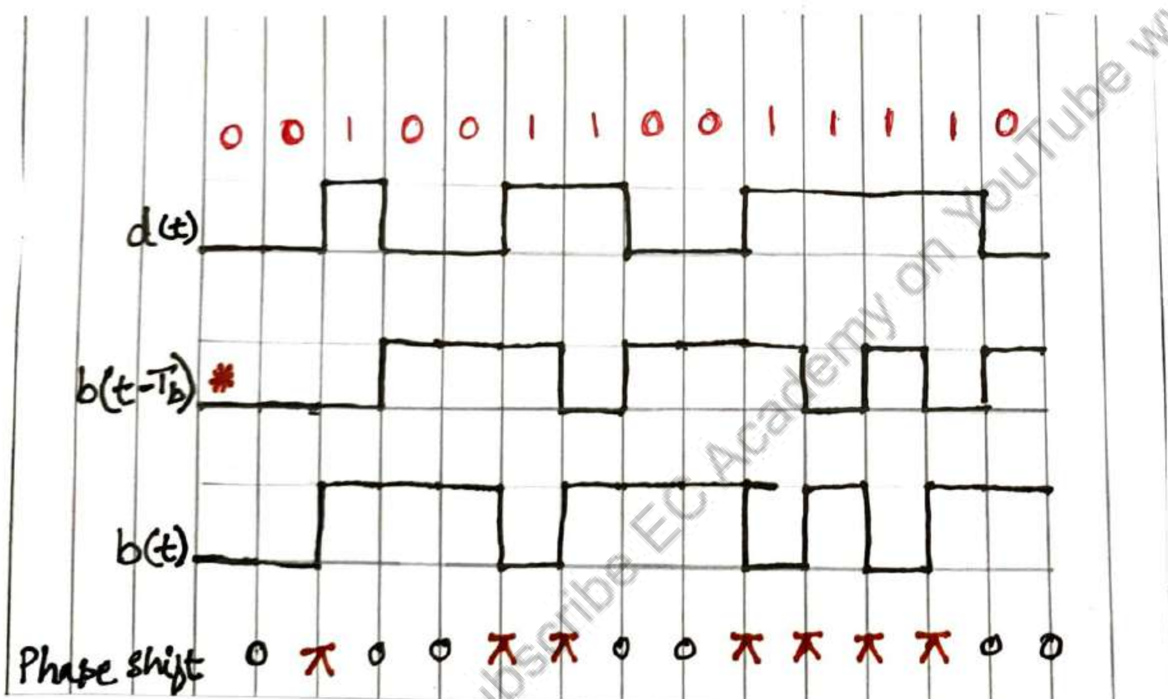
$$\boxed{BW = 2f_b}$$

BW of BPSK Signal is twice of the max frq in base band signal.

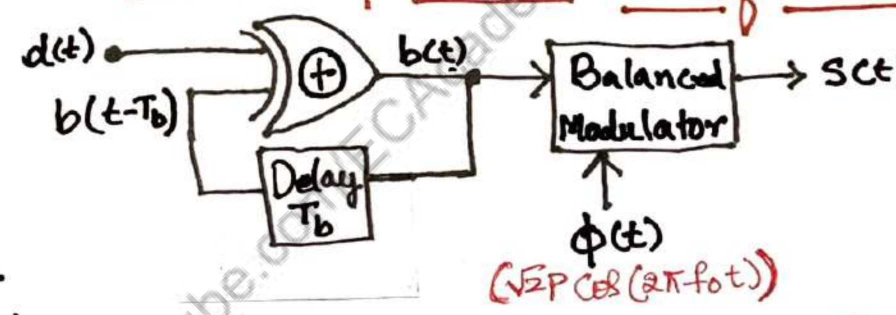
Generation of DPSK Signal:

Differential Phase Shift Keying (DPSK)

- → It is differentially coherent modulation method.
- → It does not need a synchronous carrier at demodulator.
- → Input sequance is modified such that the next bit depends on the previous bit.
- → Therefore at receiver the previous received bit are used to detect the present bit.



Generation | Transmission of DPSK



$$b(t) = d(t) \oplus b(t-T_b) \longrightarrow (1)$$

$d(t)$	$b(t-T_b)$	$b(t)$
0	0	0
0	1	1
1	0	1
1	1	0

∴ Symbol duration (T) = Duration of two bits $(2T_b)$

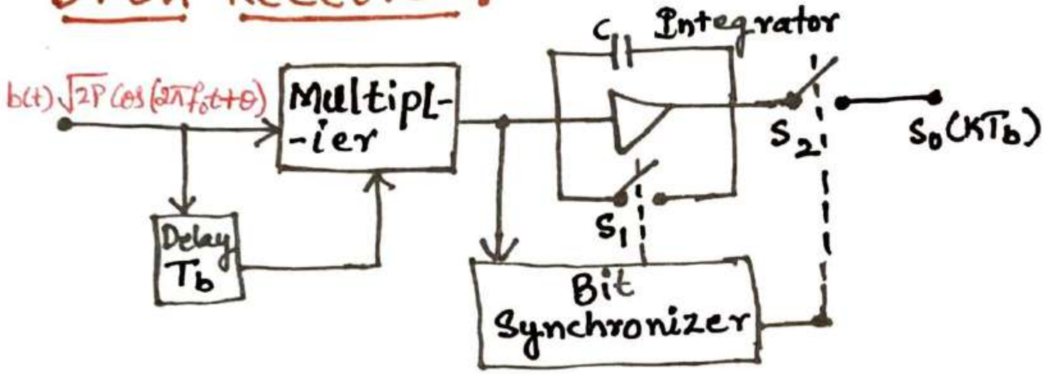
$$\Rightarrow \boxed{T = 2T_b}$$

$$\phi = \sqrt{2P}\cos(2\pi f_0 t) \longrightarrow (2)$$

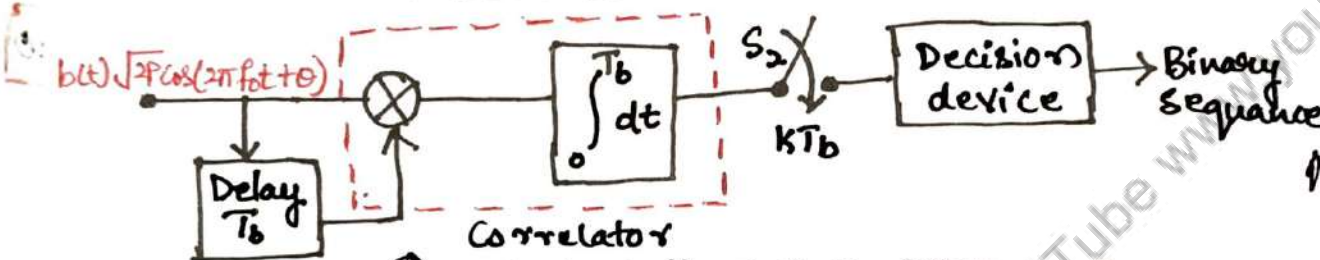
$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

$$\boxed{s(t) = \pm\sqrt{2P}\cos(2\pi f_0 t)} \rightarrow (3)$$

DPSK Receiver:



ⓐ DPSK receiver



ⓑ equivalent diagram of DPSK receiver [using correlator]

Received signal $= b(t)\sqrt{2P}\cos(2\pi f_0 t+\theta)$

Delayed signal $= b(t-T_b)\sqrt{2P}\cos(2\pi f_0(t-T_b)+\theta$

Multiplier O/P:

$$\text{Multiplier O/P} = b(t)\,b(t-T_b)\,2P\cos(\underbrace{2\pi f_0 t+\theta}_{A})\cdot\cos(\underbrace{2\pi f_0(t-T_b)+\theta}_{B})$$

$$\cos A\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$$

$$\text{Multiplier O/P} = b(t)\,b(t-T_b)\,\cancel{2}P\cdot\frac{1}{\cancel{2}}\left[\cos(2\pi \underbrace{f_0 T_b}_{n}) + \cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right]\right]$$

$f_0 \rightarrow$ Carrier frq. $\quad T_b \rightarrow$ one bit period

$\downarrow$ contains 'n' cycles of f_0

$$f_b = \frac{1}{T_b}$$

$$f_0 = n f_b \Rightarrow f_0 = \frac{n}{T_b} \Rightarrow f_0 T_b = n$$

Put $f_0 T = n$ in above eqn.

$$\text{Multiplier O/P} = b(t)\,b(t-T_b)\,P\left\{\overset{1}{\cos 2\pi n} + \cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right]\right\}$$

$$\text{Multiplier O/P} = b(t)\,b(t-T_b)\,P + b(t)\,b(t-T_b)\,P\cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right]$$

Multiplier o/p $\Rightarrow b(t)\,b(t-T_b)P + b(t)\,b(t-T_b)P\cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right]$

Integrator:- K^{th} interval,

$$s_o(KT_b) = b(KT_b)\,b((K-1)T_b)P\int_{(K-1)T_b}^{KT_b} 1\,dt + \underbrace{b(KT_b)\,b[(K-1)T_b]P\int_{(K-1)T_b}^{KT_b}\cancel{\cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right]}dt}_{0}$$

$$s_o(KT_b) = b(KT_b)\,b[(K-1)T_b]P\,\underbrace{[KT_b-(K-1)T_b]}_{T_b}$$

$$s_o(KT_b) = b(KT_b)\,b[(K-1)T_b]\,PT_b$$

$$\left.\begin{array}{l} b(t)=0(-1),\ b(t-T_b)=0(-1)\\ b(t)=1(+1),\ b(t-T_b)=1(+1)\end{array}\right\} d(t)=0$$

$$\boxed{\text{If } b(t)\,b(t-T_b)=1 \text{ then } d(t)=0}$$

$$\left.\begin{array}{l} b(t)=0(-1)\quad b(t-T_b)=1(+1)\\ b(t)=1(+1)\quad b(t-T_b)=0(-1)\end{array}\right\} d(t)=1$$

$$\boxed{\text{If } b(t)\,b(t-T_b)=-1 \text{ then } d(t)=1}$$

Decision device:

$$s_o(KT_b) = \begin{cases} -PT_b; & d(t)=1\\ +PT_b; & d(t)=0\end{cases}$$

Problem on DPSK Signal:

Binary seq. ⇒ 1 0 1 1 0 1

i) Sketch the transmitted signal.

ii) S.T. the DPSK receiver produce the original binary seq.

i) Transmitter

$d(t) \rightarrow$ input seq. $b(t) \rightarrow$ output seq.

$b(t-t_0) \rightarrow b(t)$ delayed by one bit.

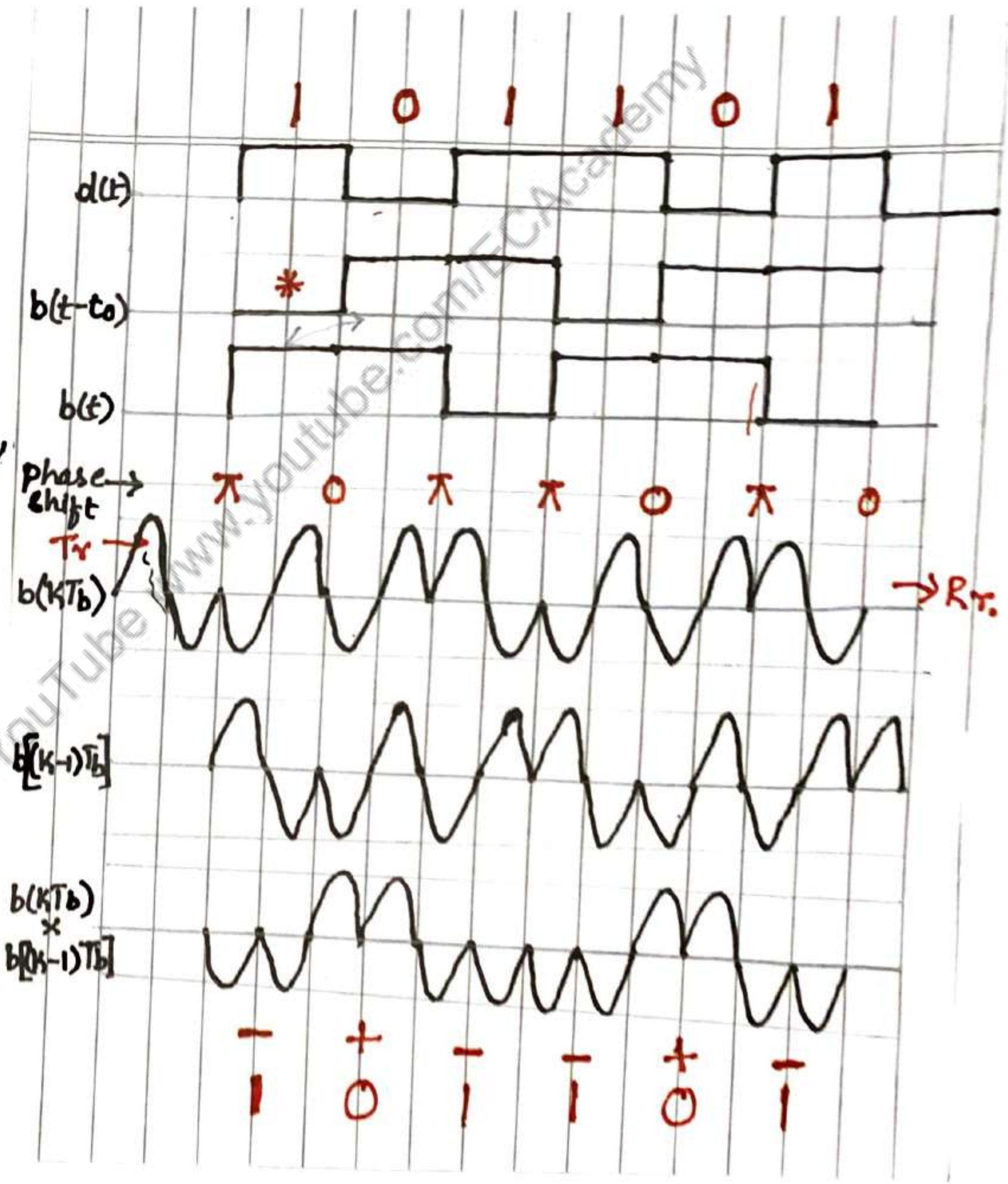
$$\boxed{b(t) = d(t) \oplus b(t-t_0)}$$

$d(t)$	$b(t-t_0)$	$b(t)$
0	0	0
0	1	1
1	0	1
1	1	0

EXOR

ii) Receiver:

$$b(KT_b)\, b[(K-1)T_b] = \begin{cases} -ve \Rightarrow 1 \\ +ve \Rightarrow 0 \end{cases}$$



Quadrature Phase Shift Keying (QPSK)

→ two Sucessive bits are grouped together ⇒ "Modulated"
→ This combination of 2 bits ⇒ 4 distinct symbols
→ when Symbol change to next symbol ⇒ phase of carrier change by $45°$ $(\pi/4)$

1	0	S_1	$\pi/4$
0	0	S_2	$3\pi/4$
0	1	S_3	$5\pi/4$
1	1	S_4	$7\pi/4$

[1 → +1V
0 → −1V]

Advantages

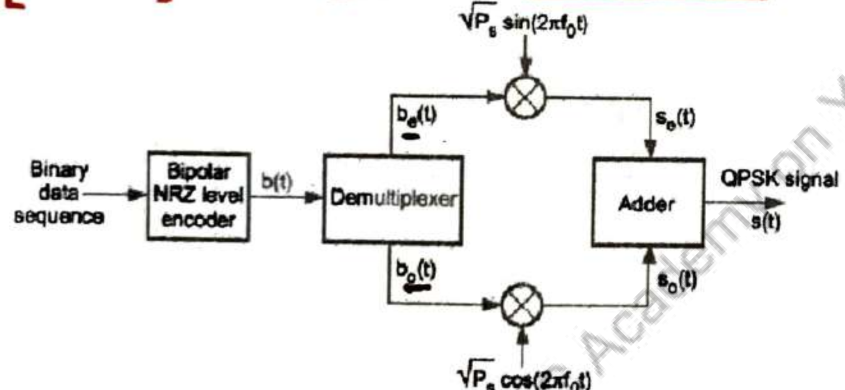
→ Signalling rate is reduced
→ Carrier freq. reduces
→ Channel BW reduces

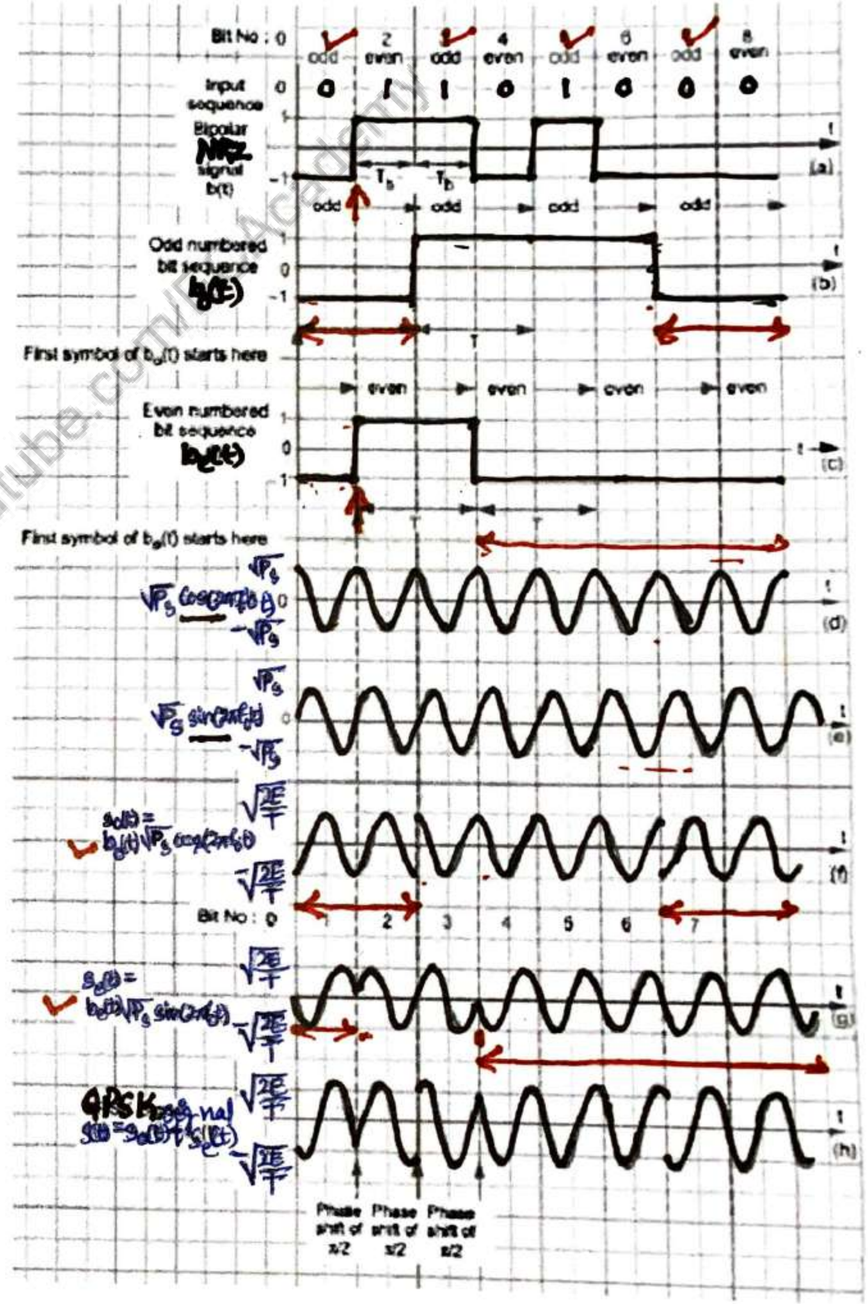
BPSK → 0 | 1
$0°$ | $180°$

QPSK $\frac{180}{4} = 45°$

$T = 2T_b$

QPSK Transmitter





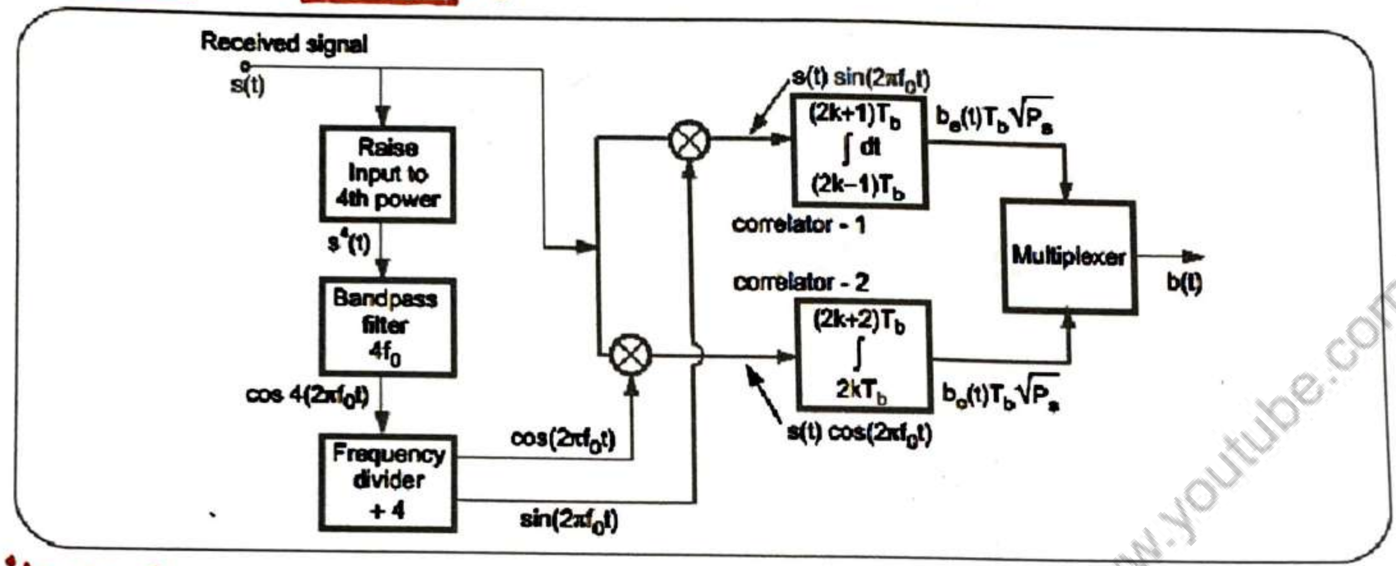
$$s_e(t) = b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

$$s_o(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t)$$

$$\boxed{s(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)}$$

QPSK Signal.

QPSK - Receiver:



$$= \frac{b_e(t)\sqrt{P_s}}{2}\Big[t\Big]_{\cancel{(2K-1)T_b}}^{(2K+1)T_b}$$

$$= \frac{b_e(t)\sqrt{P_s}}{\cancel{2}} \cdot \cancel{2}T_b$$

$$= \underline{\underline{b_e(t)\sqrt{P_s}\,T_b}}$$

$|||^{ly}$ o/p of lower integrator

$$\underline{\underline{b_o(t)\sqrt{P_s}\,T_b}}$$

i/p to integrator:

$$s(t)\sin(2\pi f_0 t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t)\sin(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin^2(2\pi f_0 t)$$

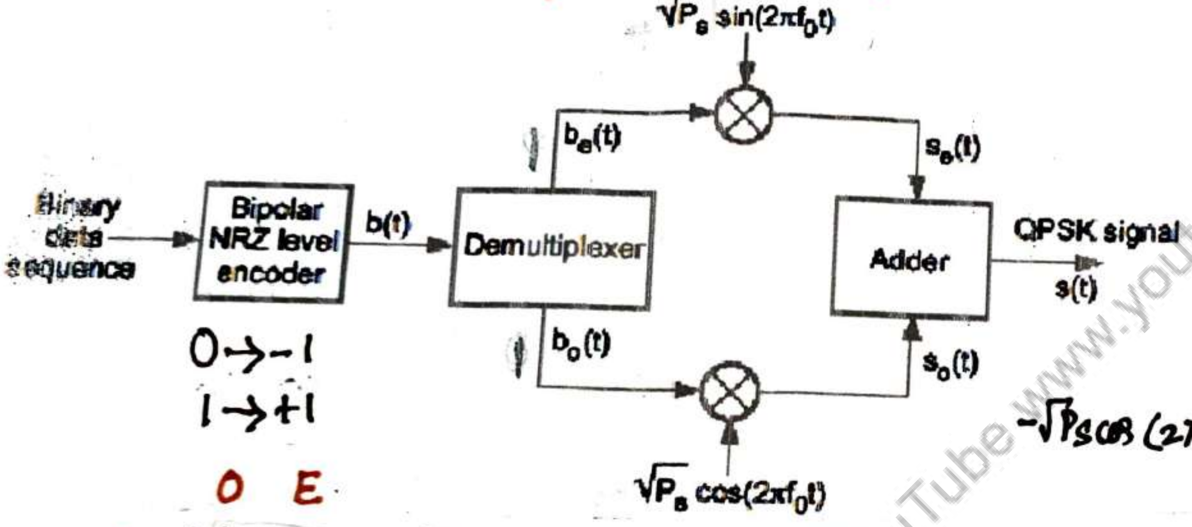
integrator o/p:

$$\int_{(2K-1)T_b}^{(2K+1)T_b} s(t)\sin(2\pi f_0 t)\,dt = b_o(t)\sqrt{P_s}\int_{(2K-1)T_b}^{(2K+1)T_b}\cos(2\pi f_0 t)\sin(2\pi f_0 t)\,dt + b_e(t)\sqrt{P_s}\int_{(2K-1)T_b}^{(2K+1)T_b}\sin^2(2\pi f_0 t)$$

$$\boxed{\frac{1}{2}\sin(2x) = \sin x\cdot\cos x \quad \& \quad \sin^2(x) = \frac{1}{2}\left[1-\cos(2x)\right]}$$

$$\int_{(2K-1)T_b}^{(2K+1)T_b} s(t)\sin(2\pi f_0 t)\,dt = \frac{b_o(t)\sqrt{P_s}}{2}\int_{(2K-1)T_b}^{(2K+1)T_b}\sin 4\pi f_0 t\,dt \xrightarrow{} 0 + \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2K-1)T_b}^{(2K+1)T_b} 1\cdot dt - \frac{b_e(t)\sqrt{P_s}}{2}\int_{(2K-1)T_b}^{(2K+1)T_b}\cos 4 f_0 t\,dt \xrightarrow{} 0.$$

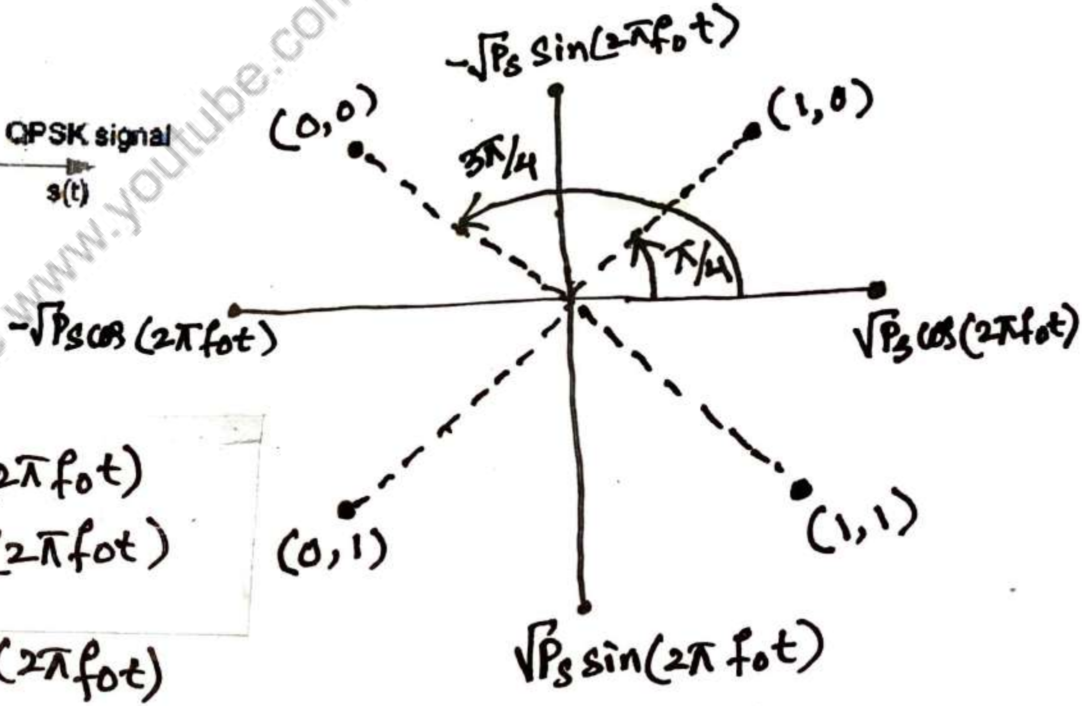
Phasor diagram of QPSK Signal



$0 \rightarrow -1$

$1 \rightarrow +1$

	O	E	
$\pi/4$	1	0	$\sqrt{P_s}\cos(2\pi f_0 t) - \sqrt{P_s}\sin(2\pi f_0 t)$
$3\pi/4$	0	0	$-\sqrt{P_s}\cos(2\pi f_0 t) - \sqrt{P_s}\sin(2\pi f_0 t)$
$5\pi/4$	0	1	$-\sqrt{P_s}\cos(2\pi f_0 t) + \sqrt{P_s}\sin(2\pi f_0 t)$
$7\pi/4$	1	1	$\sqrt{P_s}\cos(2\pi f_0 t) + \sqrt{P_s}\sin(2\pi f_0 t)$



Signal Space representation of QPSK Signal:

QPSK $\Rightarrow s(t) = s_o(t) + s_e(t)$

$$s(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

$$s(t) = \sqrt{2P_s}\cos(2\pi f_0 t)\cos\left[(2m+1)\frac{\pi}{4}\right] - \sqrt{2P_s}\sin(2\pi f_0 t)\cdot\sin\left[(2m+1)\frac{\pi}{4}\right]$$

$m = 0, 1, 2, 3$

$\times$ & $\div$ by $\sqrt{\frac{2}{T_s}}$

$$s(t) = \left\{\sqrt{P_s T_s}\cos\left[(2m+1)\frac{\pi}{4}\right]\right\}\underbrace{\sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t)}_{\phi_1(t)}$$
$$- \left\{\sqrt{P_s T_s}\sin\left[(2m+1)\frac{\pi}{4}\right]\right\}\underbrace{\sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t)}_{\phi_2(t)}.$$

$$\frac{\sqrt{2P_s}}{\sqrt{2/T_s}} = \sqrt{P_s \cdot T_s}$$

Let $b_o(t) = \sqrt{2}\cos\left[(2m+1)\frac{\pi}{4}\right]$

$b_e(t) = -\sqrt{2}\sin\left[(2m+1)\frac{\pi}{4}\right]$

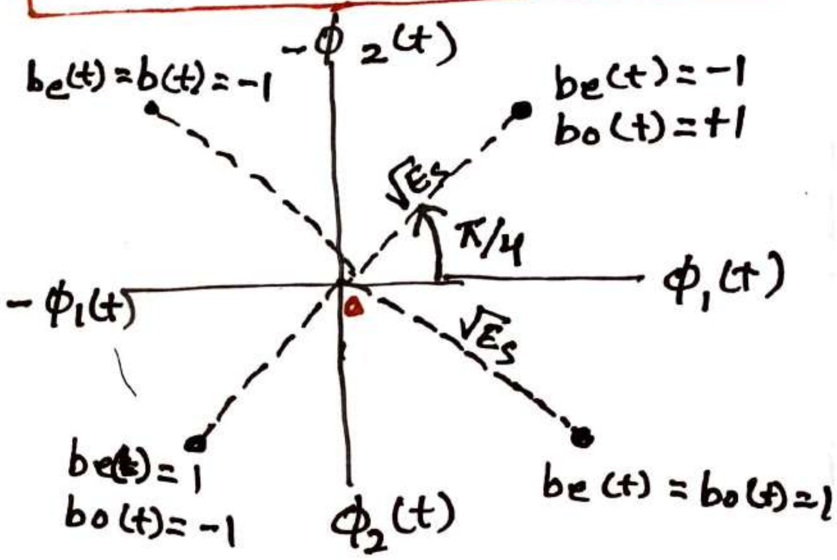
$$s(t) = \sqrt{P_s T_s}\cdot\frac{1}{\sqrt{2}}\, b_o(t)\,\phi_1(t) + \sqrt{P_s T_s}\cdot\frac{1}{\sqrt{2}}\, b_e(t)\,\phi_2(t)$$

Symbol duration $T_s = 2T_b \Rightarrow \boxed{T_b = \frac{T_s}{2}}$

$$s(t) = \sqrt{P_s T_b}\, b_o(t)\,\phi_1(t) + \sqrt{P_s T_b}\, b_e(t)\,\phi_2(t)$$

bit energy $E_b = P_s T_b$

$$\boxed{s(t) = \sqrt{E_b}\, b_o(t)\,\phi_1(t) + \sqrt{E_b}\, b_e(t)\,\phi_2(t)}$$

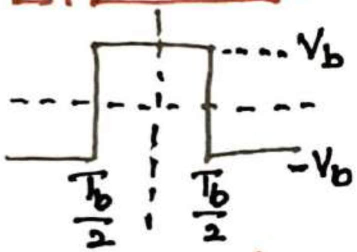


Distance of signal point from origin $\sqrt{P_s T_b + P_s T_b}$

$$= \sqrt{2P_s T_b} = \sqrt{P_s T_s} = \sqrt{E_s}$$

Spectrum & Bandwidth of QPSK Signal:

Spectrum:



NRZ pulse

PSD

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

$V_b \rightarrow$ pulse Amplitude

$T_b \rightarrow$ bit period.

$$V_b = \sqrt{P_s} \Rightarrow P_s = V_b^2$$

$$\therefore S(f) = P_s T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

PSD of Signal $b(t)$.

$b(t)$ is divided into $b_e(t)$ & $b_o(t)$

Symbol $\Rightarrow \underbrace{1 \,\&\, 0}_{\text{Equal}}$

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \longrightarrow (1)$$

$$S_o(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2 \longrightarrow (2)$$

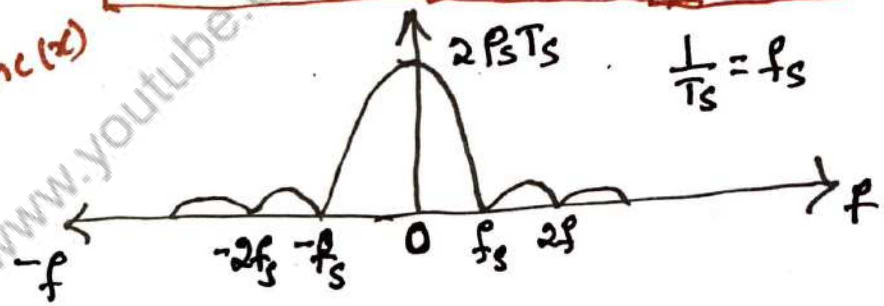
T_b by T_s ' $T_s \rightarrow$ Symbol duration.

$\frac{\sin x}{x} = \text{sinc}(x)$

$\therefore$ PSD of QPSK Signal.

$$S_B(f) = S_e(f) + S_o(f)$$

$$\boxed{S_B(f) = 2P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2}$$



Bandwidth.

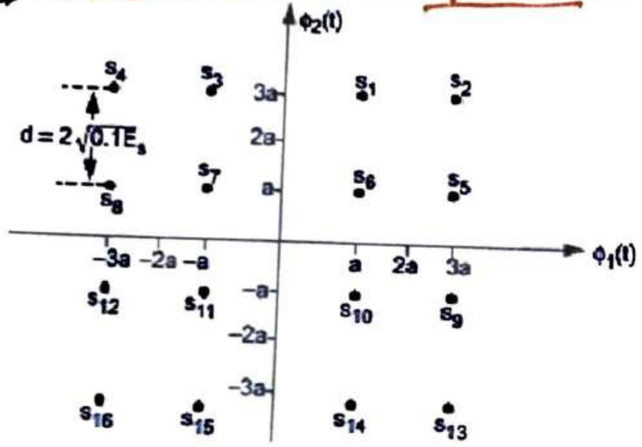
BW = Highest frq - Lowest frq in a main lobe

$$BW = f_s - (-f_s) = 2f_s$$

$$BW = \frac{2}{T_s} \quad \because T_s = 2T_b$$

$$BW = \cancel{2}/\cancel{2}T_b \Rightarrow BW = \frac{1}{T_b} \Rightarrow \boxed{BW = f_b}$$

Quadrature Amplitude Shift Keying [QASK]
OR Quadrature Amplitude Modulation [QAM]



Signal Space representation

→ Correct detection of signal → distance b/w the signal points.

→ PSK → signal points → circle.

→ Amplitude Varided → signal points will lie inside the circle

→ increase the noise immunity

Geometrical representation

→ 4bit symbols then, $2^4 = 16$ possible symbols

1011,010

The Energy of signal,

$$E_s = \frac{1}{4}\left[(a^2+a^2) + (a^2+9a^2) + (9a^2+9a^2) + (9a^2+a^2)\right]$$

$$E_s = 10a^2$$

$$a = \sqrt{0.1E_s}.$$

$$d = 2a \qquad d = 2\cdot\sqrt{0.1E_s}$$

$$\boxed{d = \sqrt{0.4\,E_s}}$$

$\therefore\ E_s = 4\cdot E_b$

$\therefore\ d = \sqrt{0.4\times 4E_b}$ $\quad \boxed{d = \sqrt{1.6E_b}}$

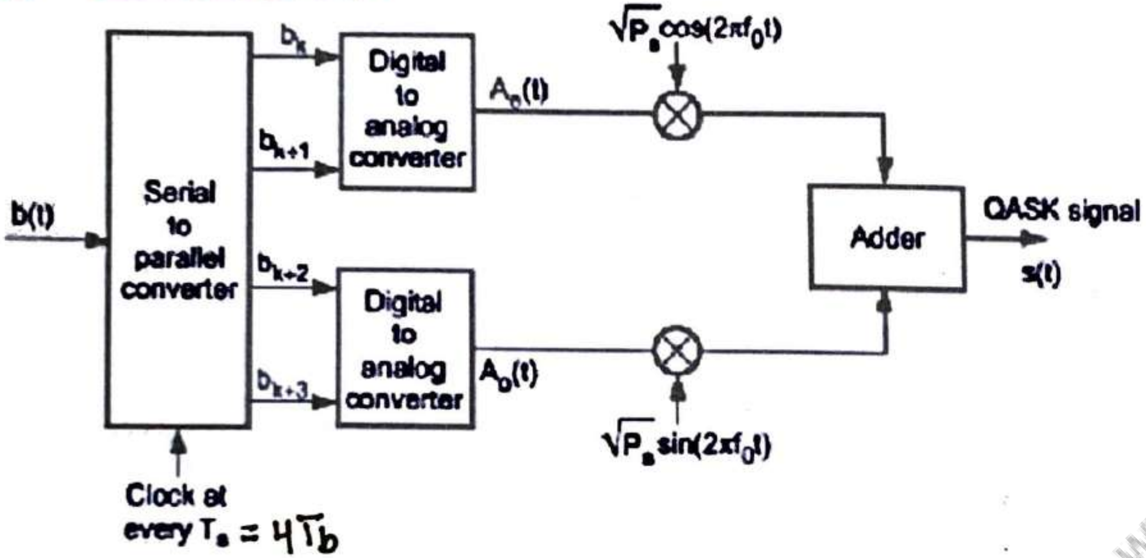
→ QPSK $\Rightarrow d = \sqrt{4E_b}$

→ PSK [16-ary] $\Rightarrow d = \sqrt{0.6E_b}$

— x —

QASK | QAM Transmission & Reception :-

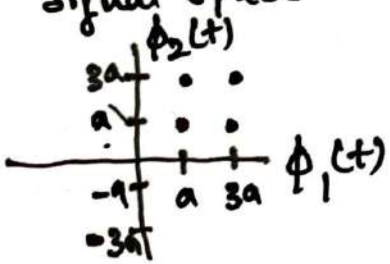
(a) Transmitter :



4bit QASK (or) 16-QASK

$$\boxed{s(t) = A_e(t)\sqrt{P_s}\cos(2\pi f_0 t) + A_o(t)\sqrt{P_s}\sin(2\pi f_0 t)} \rightarrow (1)$$

Signal space



$$s(t) = K_1 a\,\phi_1(t) + K_2 a\,\phi_2(t) \rightarrow (2)$$

K_1 & K_2 will takes ± 1, (or) ± 3

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_0 t), \quad \phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_0 t)$$

$$a = \sqrt{0.1 E_s}$$

$$(2) \Rightarrow s(t) = K_1\sqrt{0.2\frac{E_s}{T_s}}\cos(2\pi f_0 t) + K_2\sqrt{0.2\frac{E_s}{T_s}}\sin(2\pi f_0 t)$$

$$\because E_s = P_s \cdot T_s \Rightarrow P_s = \frac{E_s}{T_s}$$

$$\therefore s(t) = K_1\sqrt{0.2 P_s}\cos(2\pi f_0 t) + K_2\sqrt{0.2 P_s}\sin(2\pi f_0 t) \rightarrow (3)$$

Compare (1) & (3)

$A_e(t) = K_1\sqrt{0.2}$ (or) $K_2\sqrt{0.2}$ and $A_o(t)$

$A_e(t)$ and $A_o(t)$ $= \pm\sqrt{0.2}$ (or) $\pm 3\sqrt{0.2}$

M-ary PSK:

BPSK ⇒ 2 symbols 0, 1

$\therefore$ phase shift in BPSK $= \dfrac{2\pi}{\text{no. of symbols}} = \dfrac{2\pi}{2} = 180^\circ = \pi$

QPSK ⇒ 4 symbols

phase shift in QPSK $= \dfrac{2\pi}{4} = \dfrac{\pi}{2} = 90^\circ$

M-ary PSK

If there are "N symbols"

$2^N = M$ possible symbols

$\therefore$ phase shift $= \dfrac{2\pi}{M}$

$\therefore$ the duration of each bit will be 'NT_b'

$$\boxed{T_s = NT_b}$$

Transmitted waveform,

$$\boxed{s(t) = \sqrt{2P_s}\cos(\underbrace{2\pi f_0 t}_{B} + \underbrace{\phi_m}_{A})} \rightarrow ①$$

$m = 0, 1, 2 \cdots M-1$

$\phi_m \rightarrow$ Phase Angle $\boxed{\phi_m = (2m+1)\dfrac{\pi}{M}}$

Signal space representation.

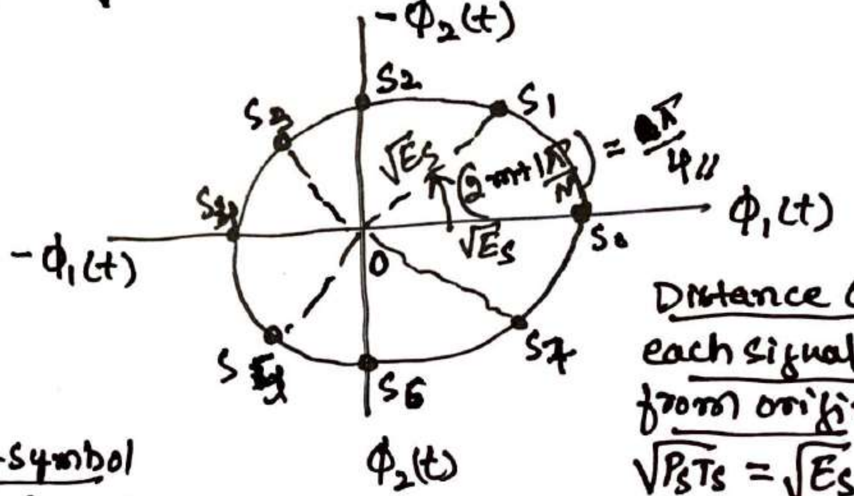
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$① \Rightarrow s(t) = \sqrt{2P_s}\cos\phi_m \cos(2\pi f_0 t) - \sqrt{2P_s}\sin\phi_m \sin(2\pi f_0 t)$

$\times$ & $\div \sqrt{2/T_s}$

$s(t) = \sqrt{P_sT_s}\sqrt{\dfrac{2}{T_s}}\cos\phi_m \underbrace{\cos(2\pi f_0 t)}_{\phi_1(t)} - \sqrt{P_sT_s}\sqrt{2/T_s}\sin\phi_m \underbrace{\sin(2\pi f_0 t)}_{\phi_2(t)}$

$s(t) = \sqrt{P_sT_s}\cos\phi_m\,\phi_1(t) - \sqrt{P_sT_s}\sin\phi_m\,\phi_2(t)$



Distance of each signal point from origin

$\sqrt{P_sT_s} = \underbrace{\sqrt{E_s}}_{\text{symbol Energy}}$

M-symbol

Signal points $s_0, s_1, s_2 \cdots s_{m-1}$

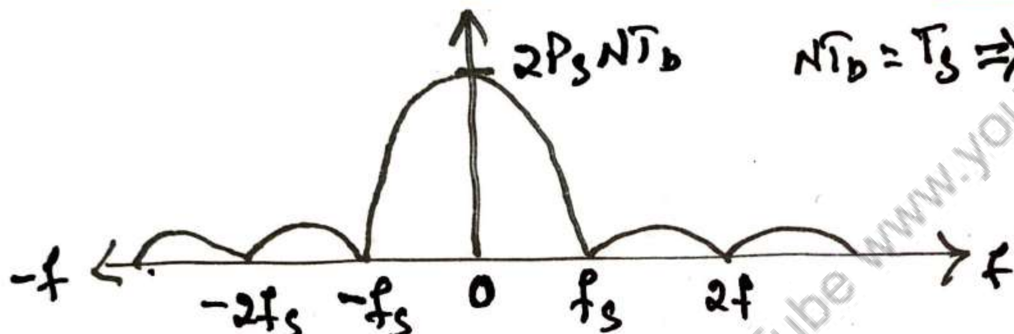
8-symbols

Signal points $s_0, s_1, \cdots s_7$

Power Spectral density

PSD of QPSK $\Rightarrow$ $S_{B(QPSK)}(f) = 2P_s T_s \left[\dfrac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$

Put $T_s = NT_b \Rightarrow$ $\boxed{S_B(f) = 2 P_s N T_b \left[\dfrac{\sin(\pi f N T_b)}{\pi f N T_b}\right]^2}$



$NT_b = T_s \Rightarrow f_s = \dfrac{1}{T_s}$

Band width

BW = HF − LF in main lobe

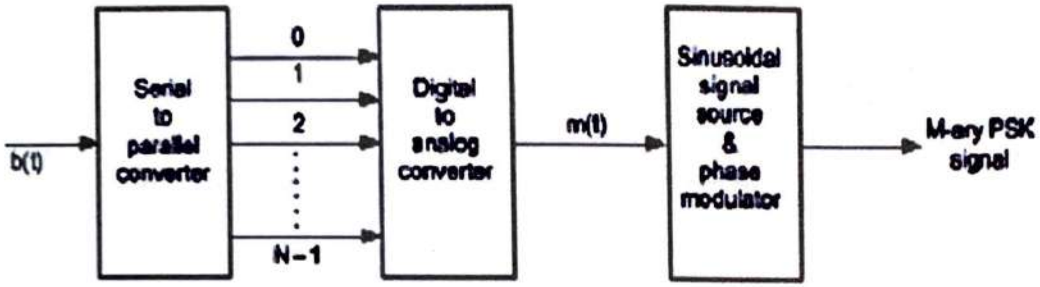
$BW = f_s - (-f_s) = 2f_s$

$BW = 2\dfrac{1}{T_s} = 2 \cdot \dfrac{1}{NT_b}$ $\qquad T_b = f_b$

$$\boxed{BW = \frac{2f_b}{N}}$$

M-ary Transmitter & Receiver:

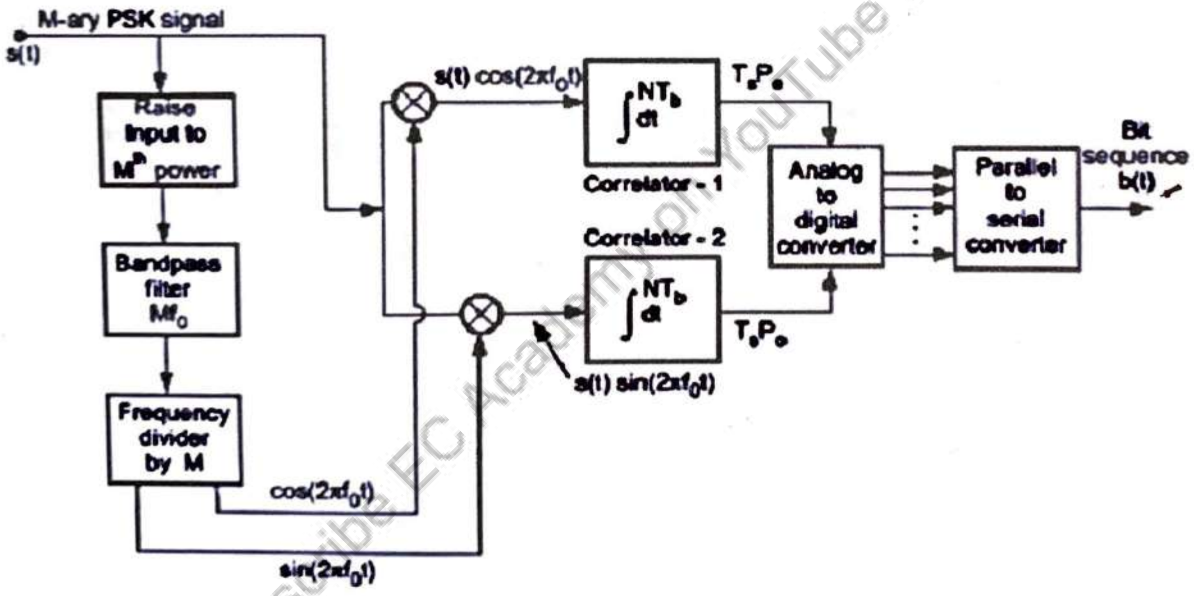
(a) Transmitter:



$\underline{NT_b}$ $\therefore m(t)$ takes

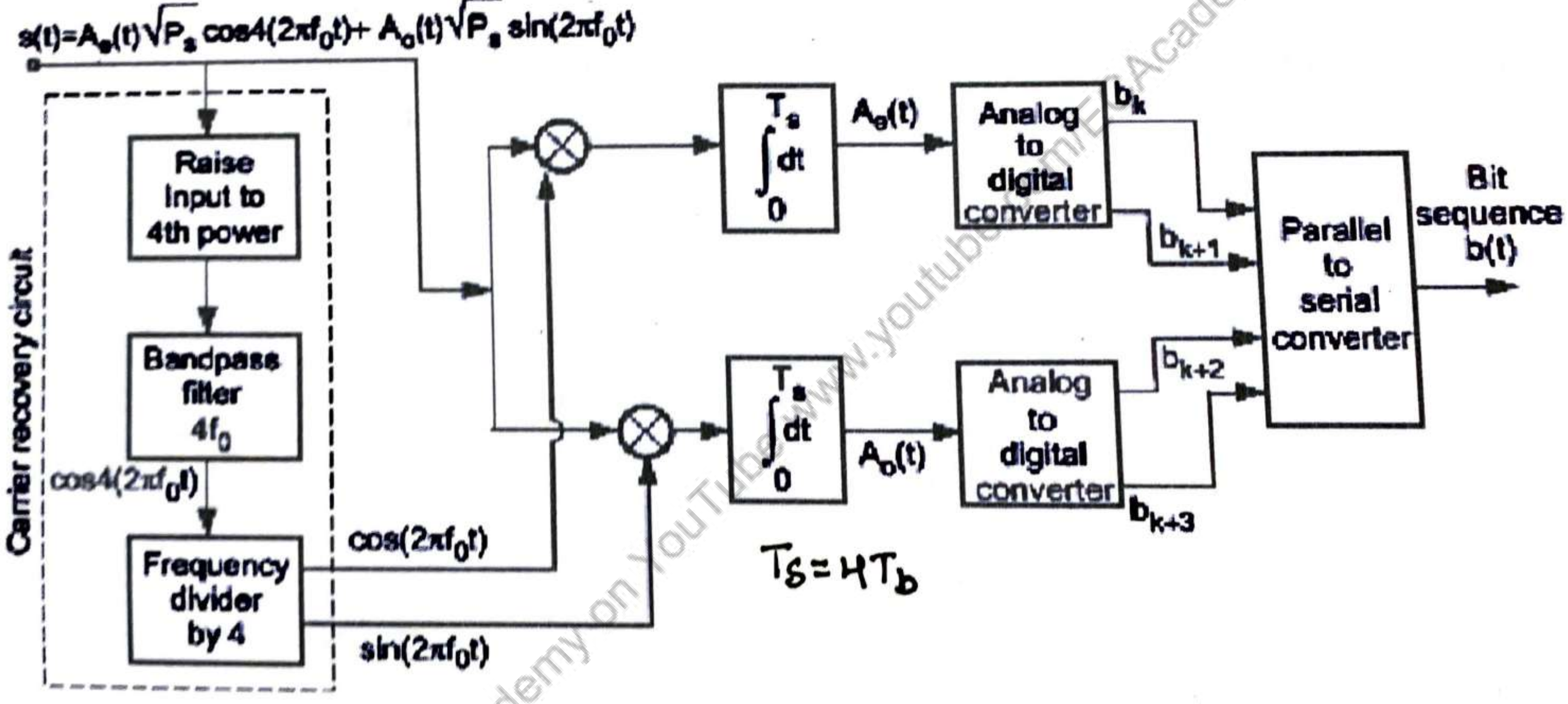
$2^N = M$ different Values.

(b) Receiver:



$T_s = NT_b$

QASK | QAM Receiver :



$T_S = 4T_b$

16-QASK (or) 4 bit QASK

Power Spectral density and Bandwidth of QASK | QAM:

PSD:

QASK,

$$s(t) = A_e(t)\sqrt{P_s}\cos(2\pi f_0 t) + A_o(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

above eqn is $\parallel^{lr}$ to M-ary PSK

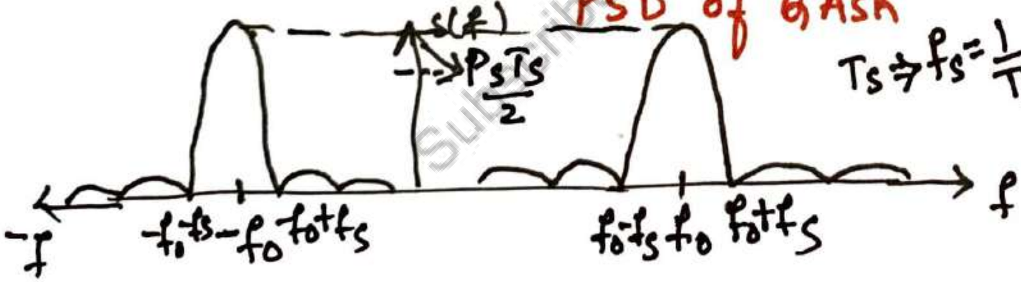
$\therefore$ PSD of QASK

$$S(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

$A_e(t)$ & $A_o(t)$

$$\therefore \boxed{S(f) = \frac{P_s T_s}{2}\left[\frac{\sin(\pi(f-f_0)T_s)}{\pi(f-f_0)T_s}\right]^2 + \frac{P_s T_s}{2}\left[\frac{\sin(\pi(f+f_0)T_s)}{\pi(f+f_0)T_s}\right]^2}$$

PSD of QASK



$T_s \Rightarrow f_s = \frac{1}{T_s}$

BW:-

BW = HF - LF in main lobe

$$BW = (f_0+f_s) - (f_0-f_s)$$

$$= \cancel{f_0} + f_s - \cancel{f_0} + f_s$$

$$= 2f_s$$

$\because f_s = \frac{1}{T_s}$

and $T_s = 'N'T_b$

$T_s = 4T_b$ $\quad T_b = \frac{1}{f_b}$.

$$T_s = \frac{N}{f_b}$$

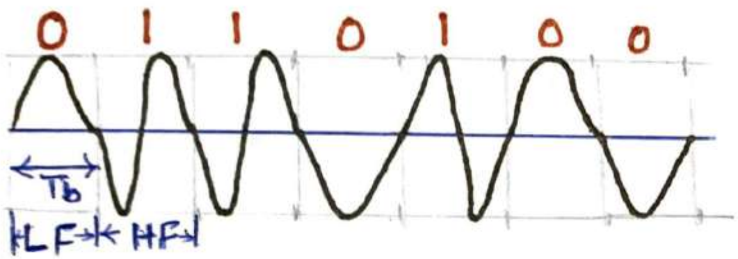
$$\therefore f_s = \frac{f_b}{N}$$

$$\boxed{B = \frac{2 \cdot f_b}{N}}$$

BW of QASK.

Binary Frequency Shift Keying [BFSK]

→ Freq of the Carrier will change according to binary symbols.



→ Hence, there are two freqies. according to binary symbols.

$b(t) = 1$; $S_H(t) = \sqrt{2P_s} \cos(2\pi f_0 + \Omega)t$ → (1)

$b(t) = 0$; $S_L(t) = \sqrt{2P_s} \cos(2\pi f_0 - \Omega)t$ → (2)

Combining eqn (1) & (2)

$$\boxed{S(t) = \sqrt{2P_s} \cos[(2\pi f_0 + d(t)\Omega)t]} \rightarrow (3)$$

Hence the carrier freqies will be,

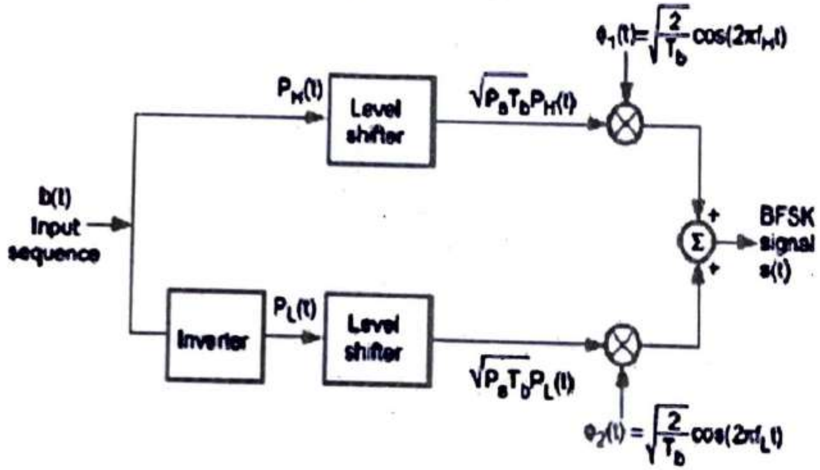
$f_H = f_0 + \left[\frac{\Omega}{2\pi}\right]$; Symbol '1'

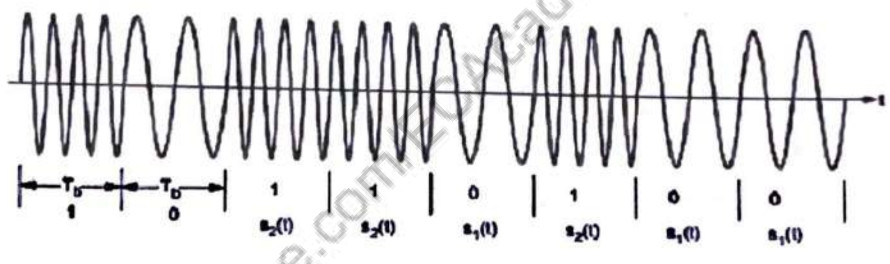
$f_L = f_0 - \left[\frac{\Omega}{2\pi}\right]$; Symbol '0'.

BPSK representation:

b(t)	d(t)	$P_H(t)$	$P_L(t)$
1	+1V	+1V	0
0	-1V	0	+1V.

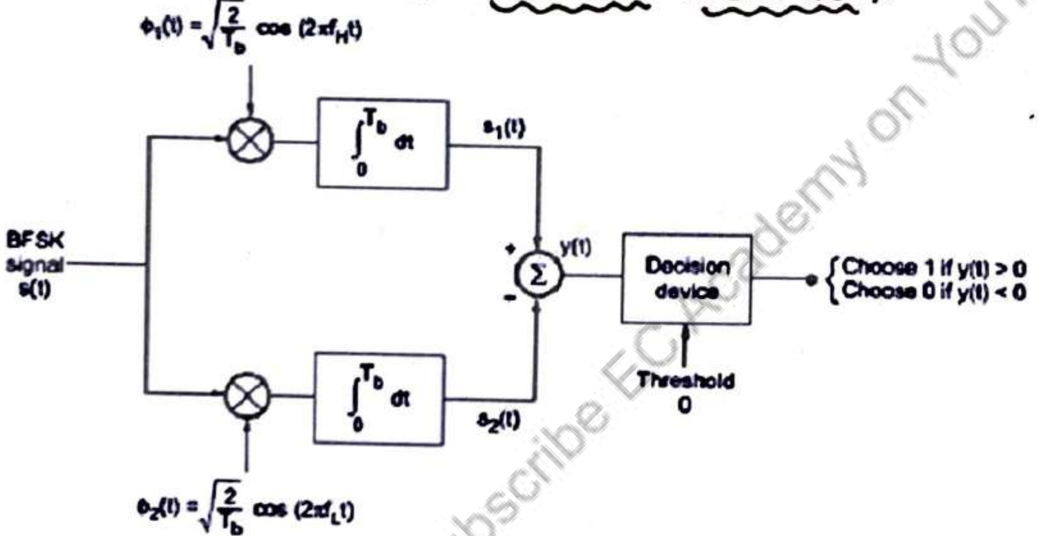
(a) BFSK Transmitter:



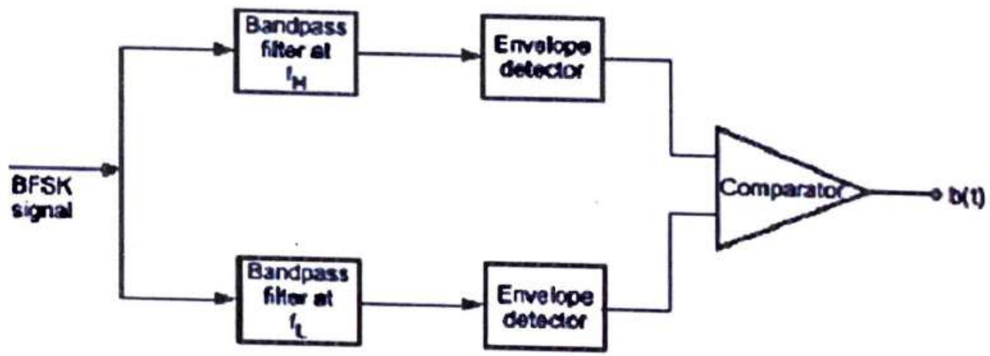


(b) BFSK Receiver:

(i) Coherent detection.



(ii) Non-coherent detection.



Spectrum and Bandwidth of BFSK:

BFSK signal,

$$s(t) = \sqrt{2P_s}\, P_H(t) \cos(2\pi f_H t) + \sqrt{2P_s}\, P_L(t) \cos(2\pi f_L t) \longrightarrow (1)$$

BPSK signal,

$$s(t) = b(t)\sqrt{2P} \cos(2\pi f_0 t) \longrightarrow (2)$$

Comparing (1) & (2)

In BPSK $b(t) \rightarrow$ bipolar.

In BFSK $P_H(t)$ & $P_L(t) \rightarrow$ Unipolar.

$\therefore$ Convert $P_H(t)$ & $P_L(t)$ in bipolar format.

$$P_H(t) = \frac{1}{2} + \frac{1}{2} P_H'(t)$$

$$P_L(t) = \frac{1}{2} + \frac{1}{2} P_L'(t)$$

$P_H'(t)$ & $P_L'(t)$ $\hookrightarrow$ bipolar (+1 or -1)

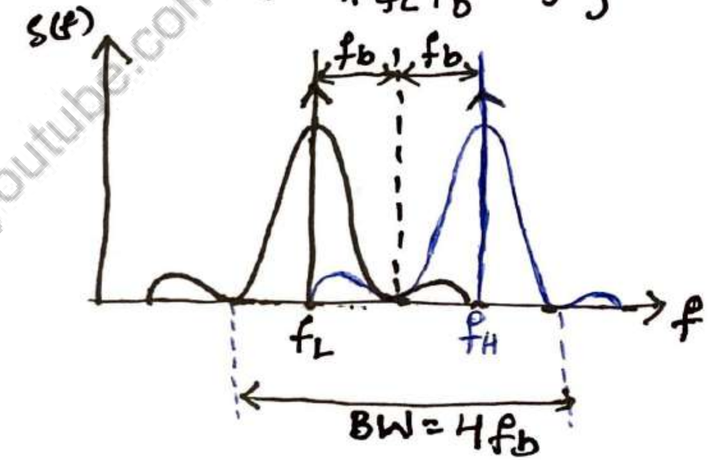
Put above values in eqn (1)

$$s(t) = \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2} P_H'(t)\right]\cos(2\pi f_H t) + \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2} P_L'(t)\right]\cos(2\pi f_L t).$$

$$\therefore s(t) = \underbrace{\sqrt{\frac{P_s}{2}}\cos(2\pi f_H t)}_{\text{impulse fun } f_H} + \underbrace{\sqrt{\frac{P_s}{2}}\cos(2\pi f_L t)}_{\text{impulse fun } f_L} + \underbrace{\sqrt{\frac{P_s}{2}}\, P_H'(t)\cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}}\, P_L'(t)\cos(2\pi f_L t)}_{\text{BPSK eqn}}$$

$\therefore$ PSD of BFSK

$$\Rightarrow S(f) = \sqrt{\frac{P_s}{2}}\left\{\delta(f - f_H) + \delta(f + f_L) + \frac{P_s T_b}{2}\left\{\frac{\sin(\pi f_H T_b)}{\pi f_H T_b}\right\}^2 + \frac{P_s T_b}{2}\left\{\frac{\sin(\pi f_L T_b)}{\pi f_L T_b}\right\}^2\right\}$$



Bandwidth :-

$$\boxed{BW = 4 f_b}$$

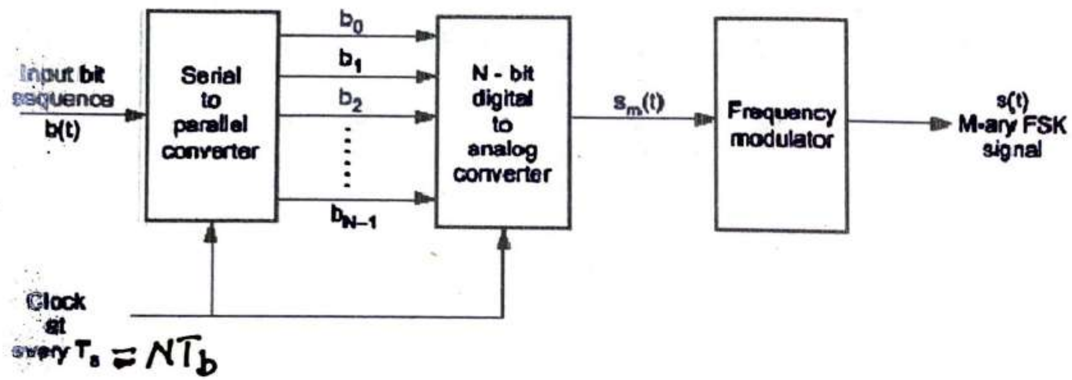
M-ary FSK [MFSK]:

BFSK → 2 symbols

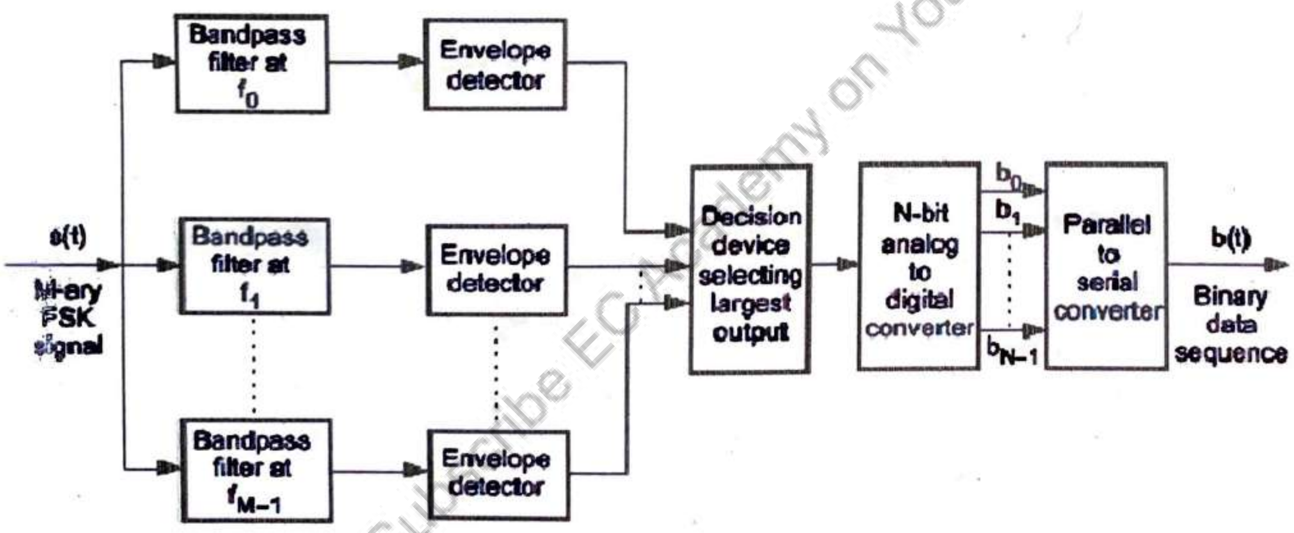
'N' bits → $2^N = M$ symbols.

$f_0, f_1, f_2 \cdots f_{m-1}$

ⓐ Transmitter:



ⓑ Receiver:



Power Spectral density:

'M' symbols $\rightarrow f_0, f_1, f_2 \cdots f_{M-1}$

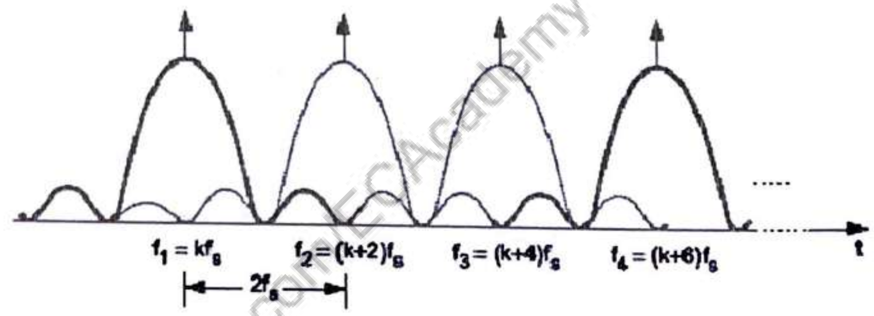
at successive even harmonics of symbol.

of frq f_s

if frq f_0 is K^{th} harmonic of symbol frq

$\therefore f_0 = K f_s$.

$f_1 = (K+2) f_s$, $f_2 = (K+4) f_s \ldots$



PSD of MFSK

$\parallel^{lr}$ to BFSK $\rightarrow$ 2 fr̲ies f_H & f_L.

$2f_b$ M fries

$2f_s$.

Band width :-

width of one mainlobe is $2f_s$.

if there are M symbols.

$M = 2^{'N'}$ $f_s = \frac{f_b}{N}$

$\therefore BW = M \times (2f_s)$

$BW = 2^N \times 2 \cdot \frac{f_b}{N} \Rightarrow \boxed{BW = \frac{2^{N+1} \cdot f_b}{N}}$

Minimum Shift Keying [MSK]

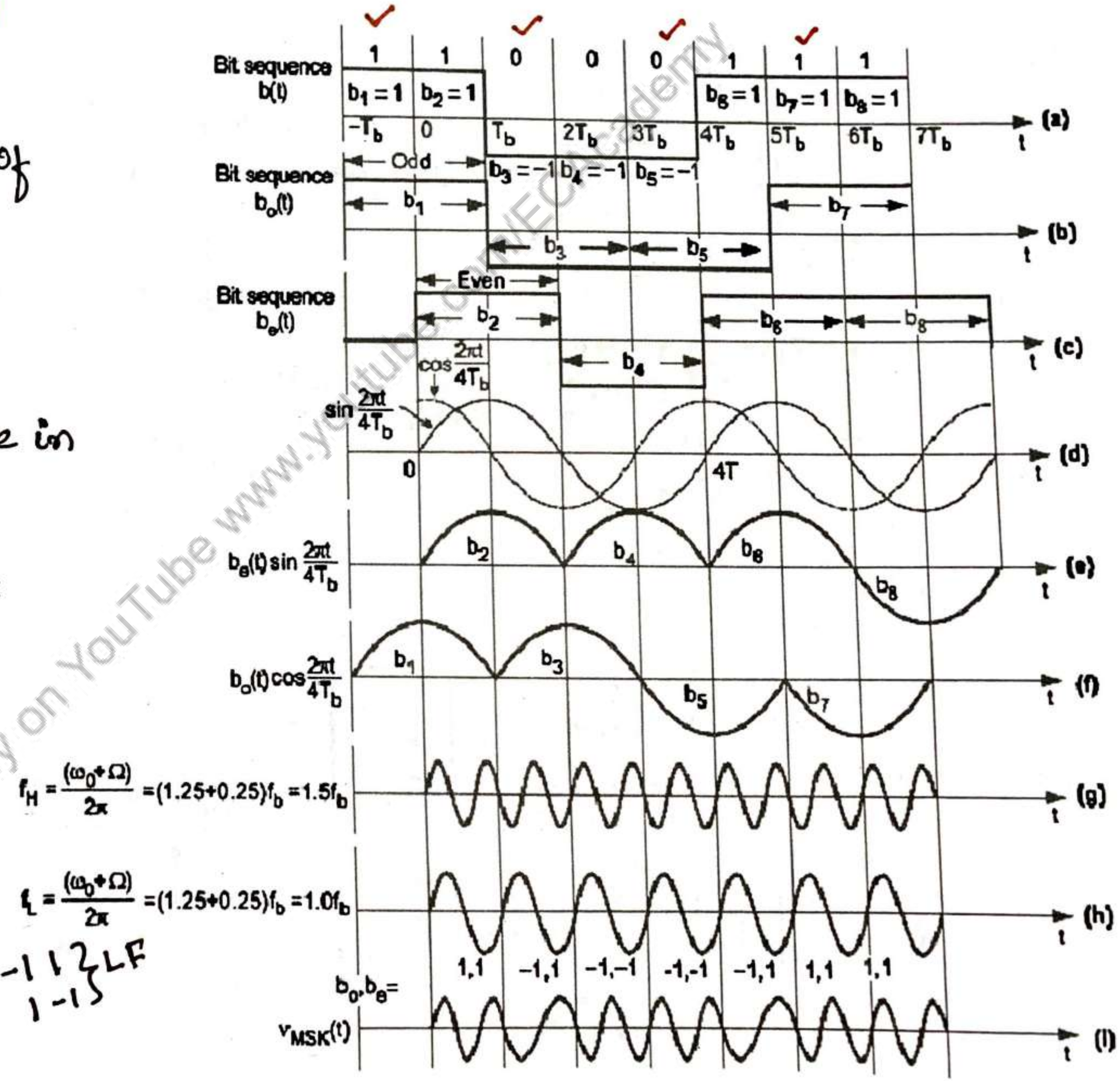
QPSK → Bandwidth is more.

Filters → alter the amplitude of waveform.

MSK is used to overcome these Problems.

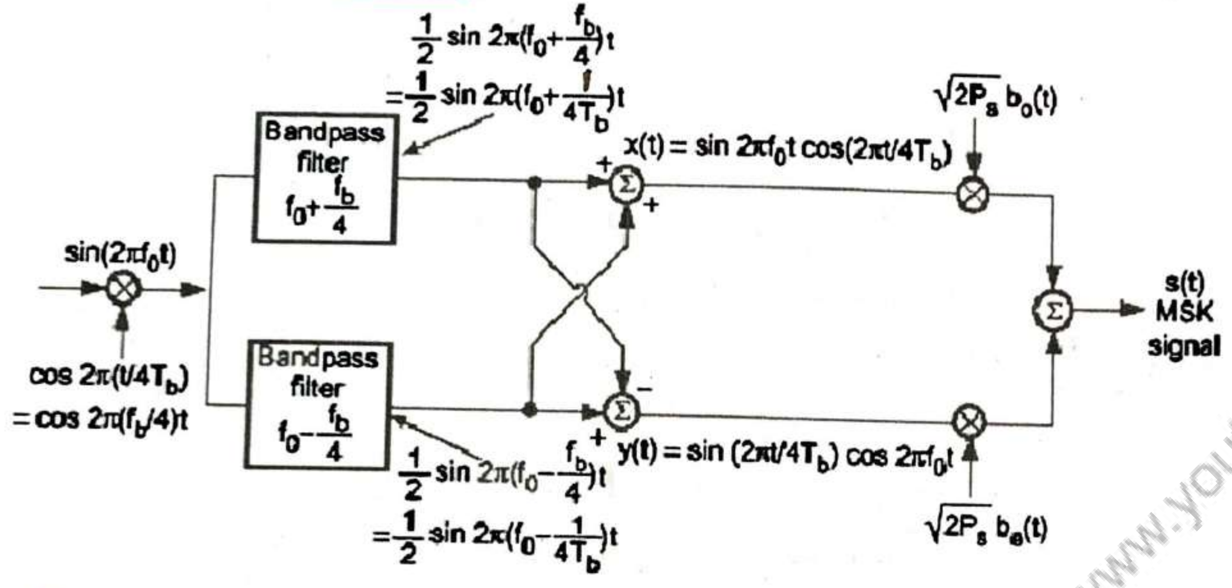
(i) There is no abrupt change in amplitude.

(ii) Bandpass filters are not required. [at receiver]



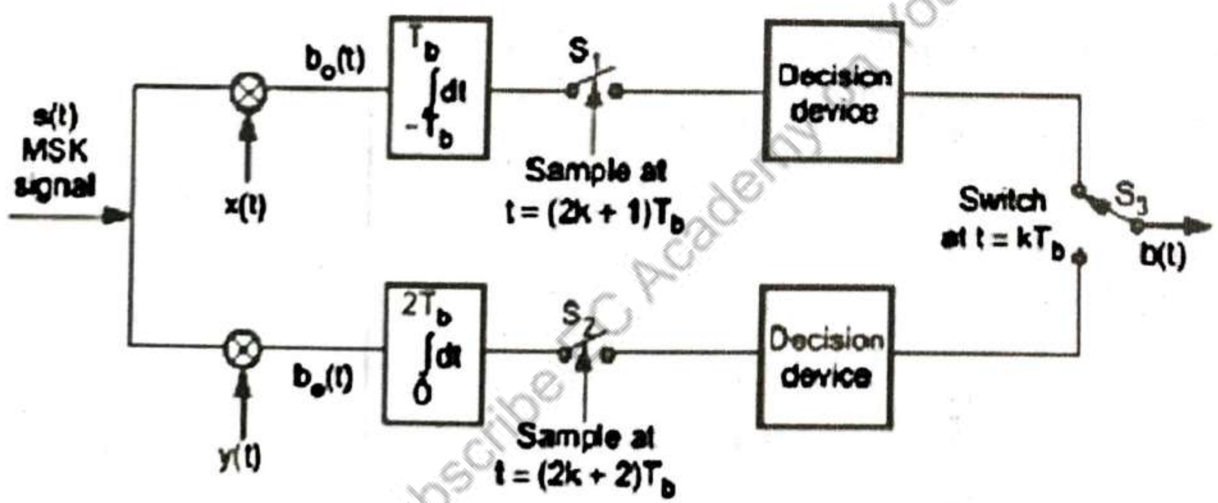
$\left.\begin{matrix}1\,1\\-1\,-1\end{matrix}\right\}$ HF $\left.\begin{matrix}-1\,1\\1\,-1\end{matrix}\right\}$ LF

(a) Transmitter:

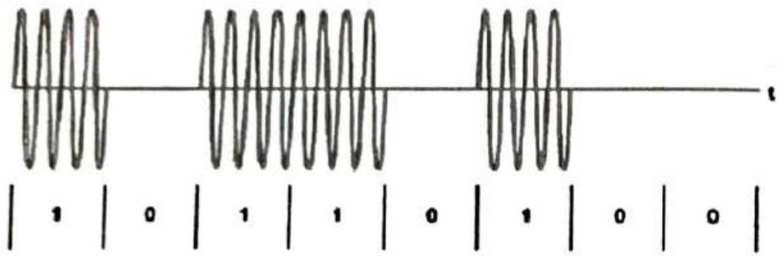


$$s(t) = \sqrt{2P_s}\left[b_e(t)\sin\left(\frac{2\pi t}{4T_b}\right)\right]\cos(2\pi f_0 t) + \sqrt{2P_s}\left[b_o(t)\cos\left(\frac{2\pi t}{4T_b}\right)\right]\sin(2\pi f_0 t)$$

(b) Receiver:



Amplitude Shift ON-OFF Keying [OOK]:



ASK waveform

→ Simple Modulation technique.

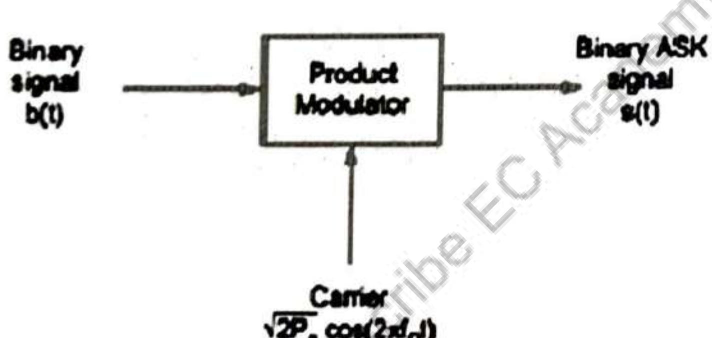
→ $\boxed{s(t) = \sqrt{2P_s}\cos(2\pi f_0 t)}$

→ symbol '1': s(t) is transmitted.
symbol '0': $s(t) = 0$

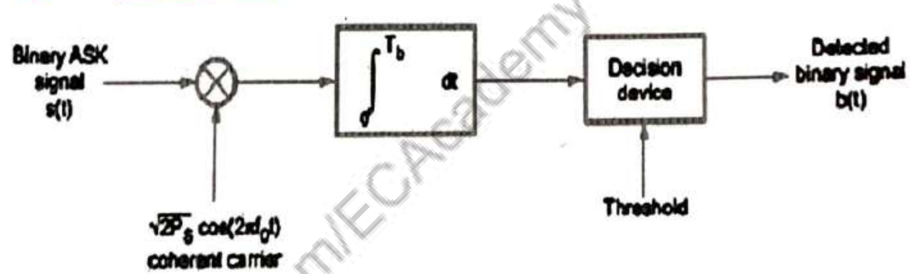
→ ON-OFF of signal. [OOK signal]

→ ASK Signal.

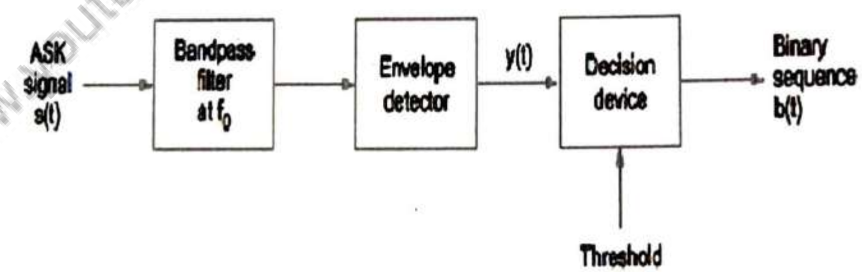
ⓐ Generation:



ⓑ Detection:– (i) Coherent detection



(ii) Non-Coherent detection



Signal space representation:

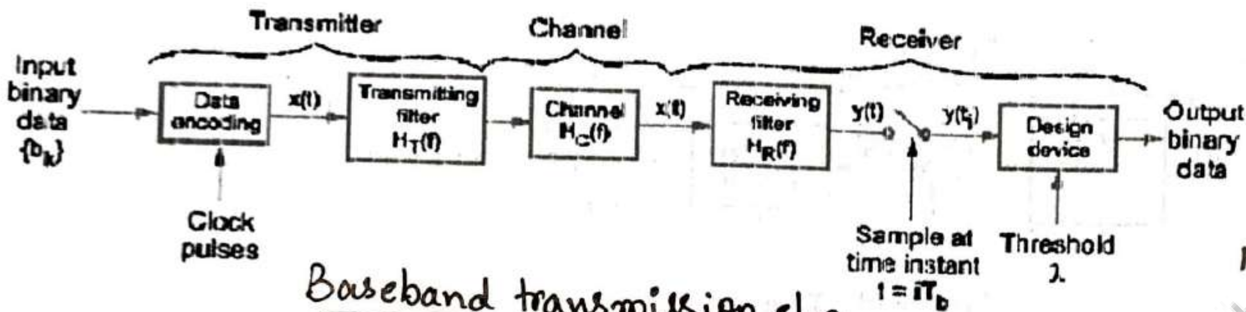
Symbol '1': $s(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_0 t)$

$$\boxed{s(t) = \sqrt{P_s T_b}\,\phi_1(t)}$$

distance b/w two symbols

$d = \sqrt{P_s T_b} = \boxed{d = \sqrt{E_b}}$

Communication over Bandlimited Channel:



Baseband transmission s/m

→ Communication through "Band limited channel" → "Base band transmission".

→ Bandlimited channels → Data without modulation.

→ Applications : LAN, Small n/ws, remote sensing & Sensor networks.

→ Problem : Inter Symbol Interference [ISI]

→ Corrective measures : Nyquist Criterion.

Baseband transmission :

→ PAM [Pulse Amplitude Modulation]

→ Amplitude of pulse varies according to i/p data.

→ Two binary levels → Symbol '0' & Symbol '1'.

→ These signals [PAM] → without modulation.

$\{b_K\}$ → i/p binary data.

Data Encoder → pulse waveform

$$\boxed{x(t) = \sum_{K=-\infty}^{\infty} A_K\, g(t - KT_b)} \longrightarrow (1)$$

Modulating Signal.

T_b → bit duration

$g(t)$ → Shaping pulse

$$A_K = \begin{cases} +a \; ; b_K = 1 \\ -a \; ; b_K = 0 \end{cases} \longrightarrow (2)$$

Transmitting Filter: → T.F. of $\boxed{H_T(f)}$

Channel → T.F. $\boxed{H_C(f)}$

Receiving Filter → T.F. $\boxed{H_R(f)}$

↳ o/p → $y(t)$ → noise replica of $x(t)$.

$y(t)$ → Sampled → $t = iT_b$

↳ $y(t_i)$ → decision device

Decision: if $y(t_i) > \lambda$ → Symbol '1'

if $y(t_i) < \lambda$ → Symbol '0'

Inter Symbol Interference [ISI]

Receiver of PAM transmission.

$$x(t) = \sum_{K=-\infty}^{\infty} A_K\, g(t - KT_b) \rightarrow (1)$$

$g(t) \rightarrow$ Shaping pulse $\quad A_K = \begin{cases} +a; & b_K = 1 \\ -a; & b_K = 0 \end{cases}$

$$\boxed{y(t) = \mu \sum_{K=-\infty}^{\infty} A_K\, P(t - KT_b)} \rightarrow (2)$$

$\mu \rightarrow$ Scaling factor & $P(t) \rightarrow$ shaping difference from $g(t)$

$\rightarrow A_K g(t) \rightarrow$ i/p applied to Transmitting Filter, channel & receiving Filter

$A_K P(t) \rightarrow$ o/p of Receiving Filter.

$\rightarrow$ Let F.T. of $g(t) \rightarrow G(f)$ & $P(t) \rightarrow P(f)$ then, [Freq. domain]

$$\mu \cancel{A_K} P(f) = H(f) A_K G(f) \rightarrow (3)$$

$H(f) \rightarrow$ Combined transfer fun.

$$\therefore\ H(f) = H_T(f) H_C(f) H_R(f) \rightarrow (4)$$

Put (4) in (3)

$$\mu P(f) = H_T(f) H_C(f) H_R(f) G(f) \rightarrow (5)$$

The o/p of R.F. is sampled at $t_i = iT_b$

$$(2) \Rightarrow y(t_i) = \mu \sum_{K=-\infty}^{\infty} A_K P(iT_b - KT_b)$$

$$y(t_i) = \mu \sum_{K=-\infty}^{\infty} A_K P[(i-K)T_b] \rightarrow (6)$$

Rearranging.

$$y(t_i) = \underbrace{\mu A_i P(0)}_{y(t_i)\ \text{when}\ i=K} + \mu \sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} A_K P[(i-K)T_b] \rightarrow (7)$$

if $P(t)$ is normalized, $P(0) = 1$

$$\therefore\ y(t_i) = \mu A_i + \mu \sum_{\substack{K=-\infty \\ K \neq i}}^{\infty} A_K P[(i-K)T_b] \rightarrow (8)$$

$i = 0, \pm1, \pm2, \pm3 \ldots$

(i) 1st term: is contribution from ith transmitted bit.

(ii) 2nd term: effect of all other bits transmitted before and after sampling instant 't_i'.

ISI: is the presence of effect of other bits interference with o/p of required bit.

Let us consider eqn (8)

$y(t_i) = \mu A_i$ at $t = iT_b$

$\boxed{A_i \rightarrow \text{Correct bit}}$

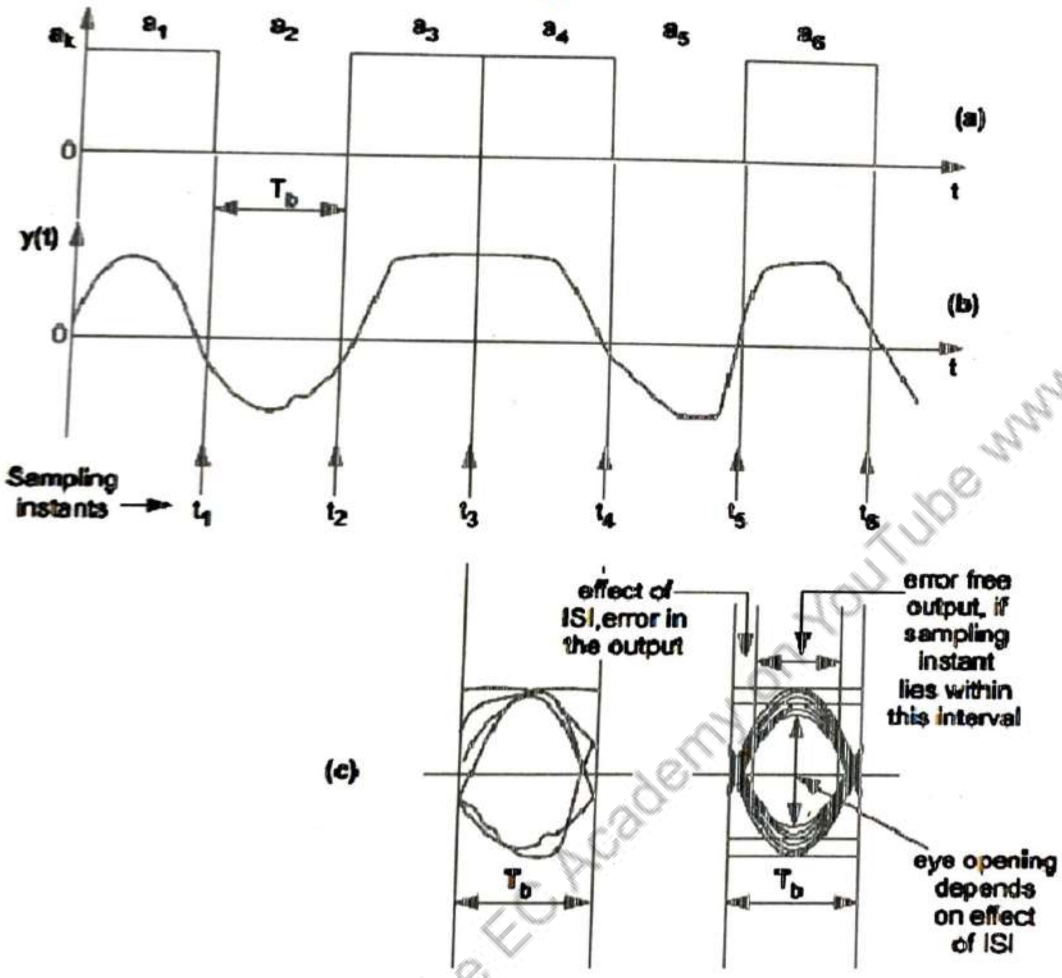
2nd term is entirely ISI

$\Rightarrow$ Elimination of ISI:

(i) proper design of pulse spectrum [$G(f)$], Transmitting Filter [$H_T(f)$], Received Filter [$H_R(f)$] & Channel [$H_C(f)$]

(ii) Individual spectrum of the pulse should be seperated by a bit period [T_b].

Baseband Transmission of M-ary data & Eye Diagram.



Eye Diagram

M-ary Transmission → M level (or) Amplitude of wave forms

DE → data → PAM Signal.

Ex:- 2 bits → 4 symbols $M=4$

$T = 2T_b$

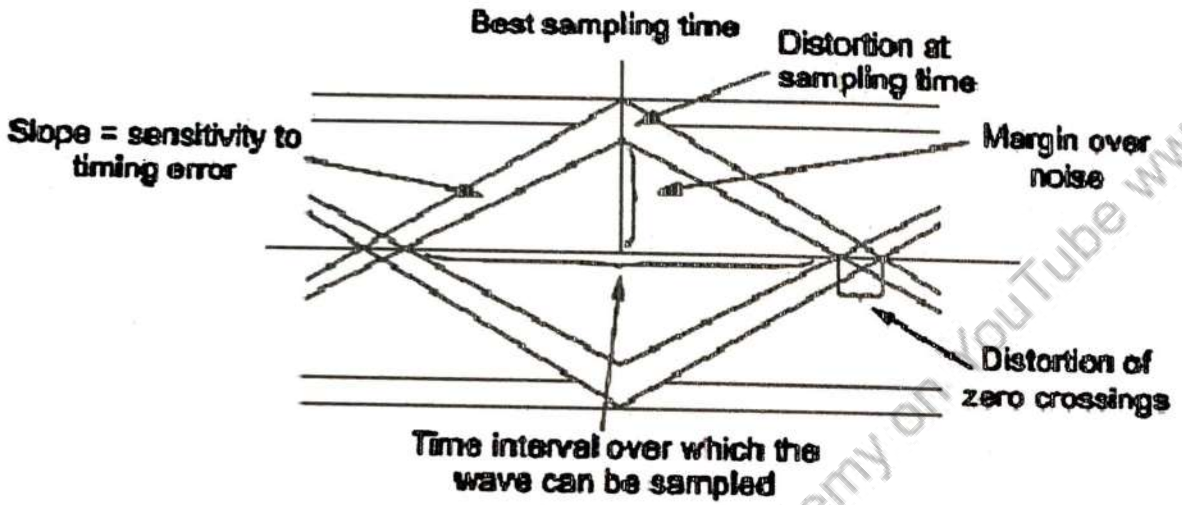
M-Symbols → $\log_2 M$ bits are combined

∴ T = one bit period × No of bits / sy.

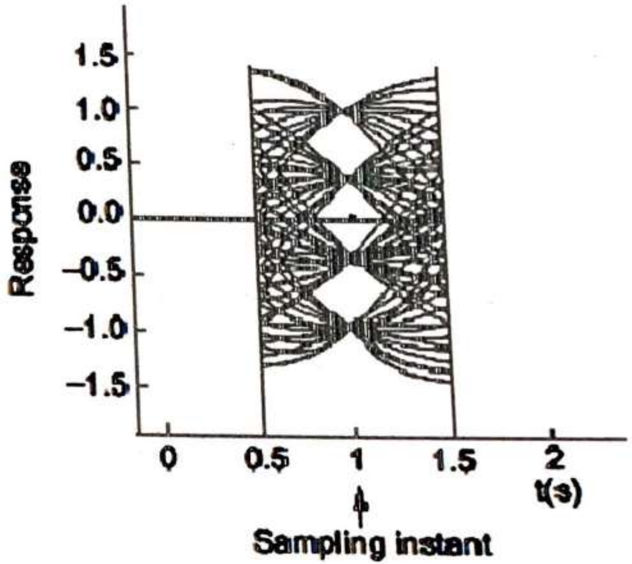
$\boxed{T = T_b \cdot \log_2 M}$ Power ↑

Eye diagram → Effect of ISI

Interpretation of Eye pattern:



M=2



M=4

Band limited Ideal channel with Zero ISI:

Nyquist pulse shaping criterion: (i) Time domain

$$y(t_i) = \mu A_i + \mu \underbrace{\sum_{\substack{K=-\infty \\ K\neq i}}^{\infty} A_K P[(i-K)T_b]}_{\text{Effect of ISI}} \longrightarrow ①$$

→ The second term → ZERO

→ P(t) is controlled $\boxed{P[(i-K)T_b] = \begin{cases} 1 ; & i=K \\ 0 ; & i\neq K \end{cases}} \rightarrow ②$

Condition in time domain

$$\boxed{y(t_i) = \mu A_i}$$

(ii) Frequency domain

→ Fourier spectrum of P(t)

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - nT_b) \longrightarrow ③$$

$f_b \rightarrow$ Sampling frq

$P_\delta(f) \rightarrow$ spectrum of $P(nT_b)$

$P(f) \rightarrow$ spectrum of $P(t)$

$P(nT_b) \rightarrow$ Finate length impulses

Period $\rightarrow T_b$ & amplitude $\rightarrow P(t)$

$$\therefore P_\delta(t) = \sum_{n=-\infty}^{\infty} P(nT_b)\,\delta(t-nT_b)$$

F.T.

$$\therefore P_\delta(f) = \int_{-\infty}^{\infty} P_\delta(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} P(nT_b)\,\delta(t-nT_b)\right] e^{-j2\pi ft}\, dt.$$

Let $n = i-K$

$$\therefore P_\delta(f) = \int_{-\infty}^{\infty} \sum_{K=-\infty}^{\infty} P[(i-K)T_b]\ \delta[t-(i-K)T_b]\, e^{-j2\pi ft}\, dt. \longrightarrow ④$$

use eqn ④ in ②

$$\therefore P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} P(0)\,\delta(t)\,e^{-j2\pi ft}\,dt ; & i=K \\ \int_{-\infty}^{\infty} 0\ \delta(t)\,e^{-j2\pi ft}\,dt ; & i\neq K. \end{cases}$$

$$\Rightarrow P_\delta(f) = \int_{-\infty}^{\infty} P(0)\,\delta(t)\,e^{-j2\pi ft}\,dt \quad ; i=K$$

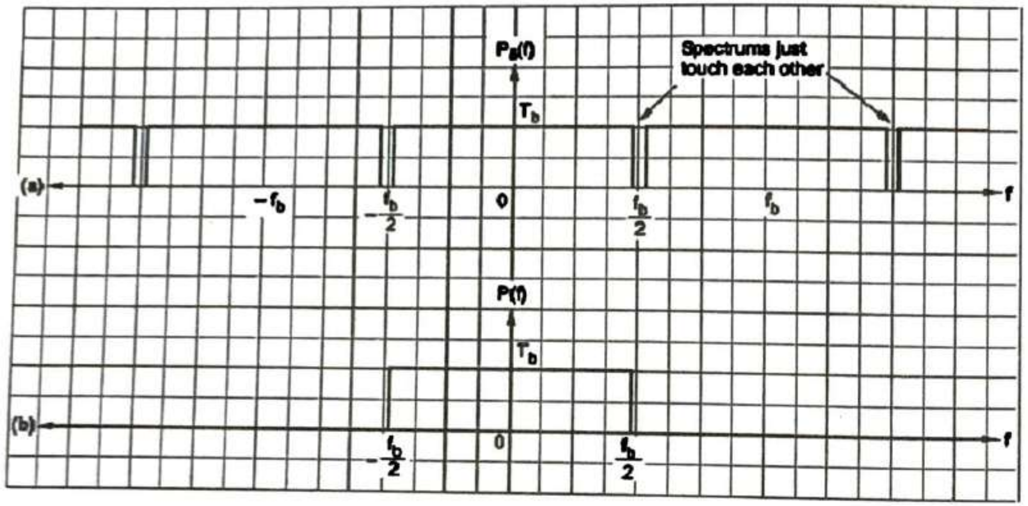
$$\therefore P_\delta(f) = P(0)\int_{-\infty}^{\infty} \delta(t)\,\underbrace{e^{-j2\pi f t}}_{1}\,dt.$$

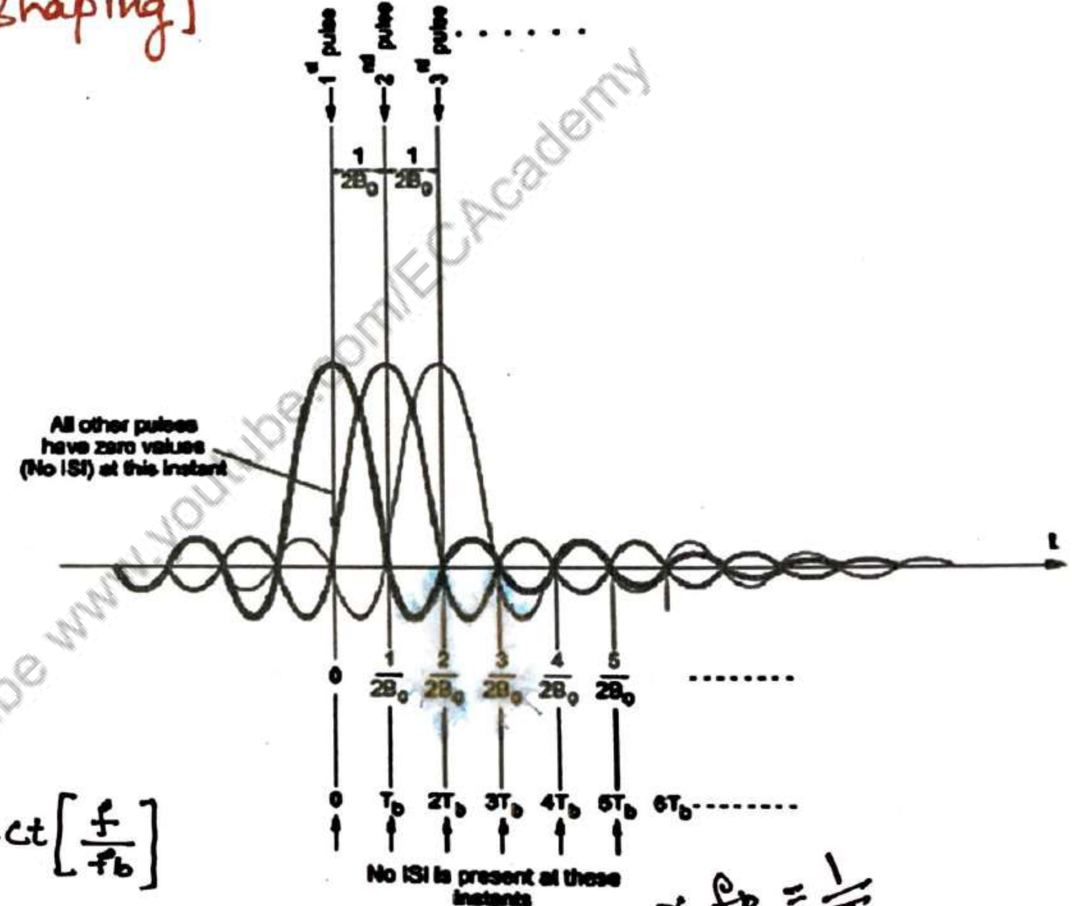
$$\therefore P_\delta(f) = P(0) \ ; i=K \longrightarrow ⑤ \Rightarrow P_\delta(f) = 1$$

$$③ \Rightarrow 1 = f_b \sum_{n=-\infty}^{\infty} P(f-nf_b) \Rightarrow \boxed{T_b = \sum_{n=-\infty}^{\infty} P(f-nf_b)}$$

Condition in Frq domain.

Ideal Nyquist Channel: [sinc pulse shaping]





$$T_b = \sum_{n=-\infty}^{\infty} P(f - n f_b)$$

$$P(f) = \frac{1}{f_b}\,\text{rect.}\left[\frac{f}{f_b}\right]$$

using std. F.T.

$$\text{sinc}(f_b t) \longleftrightarrow \frac{1}{f_b}\,\text{rect}\left[\frac{f}{f_b}\right]$$

$\therefore$ I.F.T

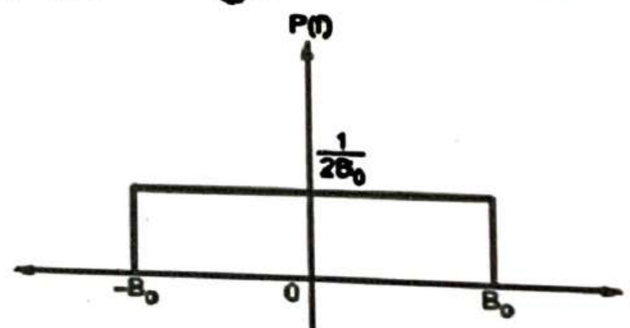
$$P(t) = \text{sinc}\,(f_b t)$$

Bit rate $= 2B_0 = 2 \cdot \frac{f_b}{2} = \frac{1}{T_b}$

BW of the pulse $B_0 = f_b/2 \Rightarrow \underline{f_b = 2B_0}$

$\therefore P(t) = \text{sinc}\,(2B_0 t) \longrightarrow (7)$

$$\boxed{P(t) = \frac{\sin\,(2\pi B_0 t)}{2\pi B_0 t}} \longrightarrow (8)$$



$B_0 \rightarrow$ Nyquist B.W. $\rightarrow$ Zero ISI.

$$P(f) = \frac{1}{2B_0}\,\text{rect.}\left[\frac{f}{2B_0}\right]$$

Detection of data by Controlled ISI

→ DUOBINARY signalling

Sample of received filter

$$\boxed{y(t_i) = b(t_i) + n(t_i)}$$

0+0 → -1-1 → -2
0+1 → -1+1 → 0
1+0 → +1-1 → 0
1+1 → +1+1 → +2

$y(t_i) = A_i + A_{i-1} + n(t_i)$

A_i → Amplitude of transmitted seq.

$A_i \rightarrow +1$ or -1

$n(t_i)$ → Guassian noise

$\therefore\ b(t_i) \rightarrow +2, -2,$ or 0

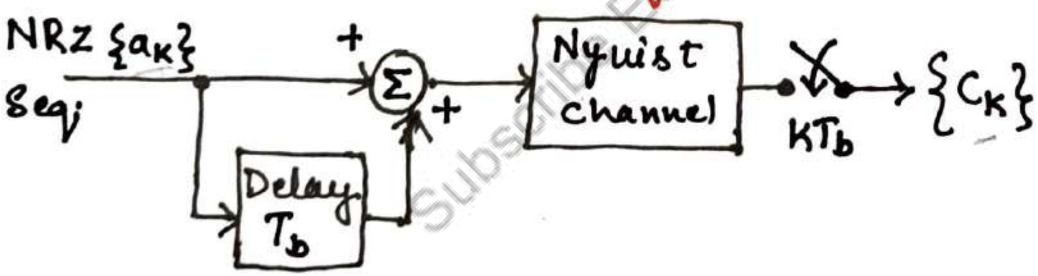
$\therefore\ \boxed{y(t_i) = A_i + A_{i-1}}$ or $\boxed{C_K = a_K + a_{K-1}}$

→ Two successive bits are used for evaluation.

→ symbol by symbol detection of data or duobinary encoding or correlative coding.

→ Signalling rate at $2B_0$ → channel BW B_0

Duobinary (DB) Encoding:



DUO → double the transmission capacity.

i/p seq. $\{b_K\}$ → 1 or 0

NRZ encoding $\begin{cases} a_K = +1 \ ; b_K = 1 \\ a_K = -1 \ ; b_K = 0 \end{cases}$

→ $\{a_K\}$ → 3 level signal i.e., +2, -2 or 0

o/p of encoder $\boxed{C_K = a_K + a_{K-1}}$

-1 + (-1) = -2

Reconstruction:

$$\boxed{\hat{a}_K = C_K - \hat{a}_{K-1}}$$

-2 - (-1) → -1

Example:

b_K	0	0	1	1	0	1	0	0
a_K	-1	-1	+1	+1	-1	+1	-1	-1
C_K	-	-2	0	+2	0	0	0	-2
$\hat{a}_K$	-1	-1	+1	+1	-1	+1	-1	-1
b_K	0	0	1	1	0	1	0	0